Eidgenössische Technische Hochschule Zürich
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Distributed Computing Group

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Ad Hoc And Sensor Networks
Sample Solution to Exercise 8

Assigned: November 9, 2009
Due: November 16, 2009

1 Dominating Sets in Unit Disk Graphs

a) The greedy algorithm picks consecutively the node that covers the most nodes that are not already dominated. In The Unit Disk Graph model, this algorithm yields an approximation ratio of 6. The proof is similar to the one showing that the MST has bounded degree in Exercise 3. Let's consider a node u that is part of the dominating set in the optimal solution. It covers all nodes, denoted by the set S_u, in the circle of radius 1. The greedy algorithm uses at most 6 nodes to cover all the nodes in S_u. The disk of u is therefore partitioned into 6 equal pieces (see Figure 1). In one such piece the greedy algorithm chooses at most one node to cover nodes in S_u. All nodes inside one such piece form a clique as the maximum distance between any two points is at most 1. Thus, all of them can be covered by one node. Note that it is possible that the greedy algorithm picks more than one node in a piece. These additional nodes can however be assigned to another node in the optimal dominating set.

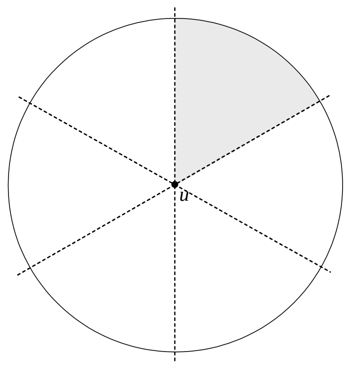

Figure 1: The greedy algorithm picks at most one node in the gray piece.

b) Surprisingly, this approach does not work. Consider the situation depicted in Figure 2. Nodes u and v have a distance of exactly one and the link (u, v) is required to connect two clusters of nodes. That is, there are no other nodes in between u and v. Reducing the grid size has no influence on the fact that the link (u, v) has to be part of any CDS. We now have to show, that the grid algorithm does not select nodes u and/or v for the constructed DS. Assume there is another node w in the same cell as u. Furthermore, $d(u, v) < d(w, v)$ and w is closer to the cell center than u. The algorithm will therefore not choose node u but w as dominator for this cell. No matter how small cell sizes are chosen there may always be a node w preventing the selection of u or v.

Please note that, it is possible to solve the problem if the positions of all nodes are known and the grid can be placed and spaced to make sure all critical nodes are in the center of their respective cells. However, with this global knowledge the problem of creating a connected dominating set becomes trivial and using the decentralized grid algorithm makes no more sense.

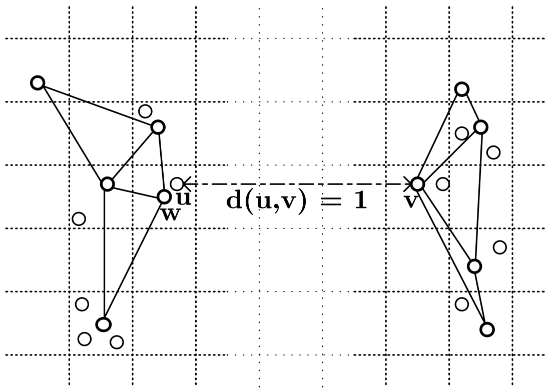

Figure 2: Counterexample why the grid algorithm for dominating sets cannot be used to construct a CDS

Connectivity Models

a) Australia's outback is hardly obstructed, very flat, and the radio signals can freely propagate in space. In such an environment, the use of more idealistic connectivity models such as the Quasi Unit Disk Graph might be justified. In contrast, in an office building, there are many walls and obstacles influencing the nodes' connectivity. Here, it is advisable to study more general models such as the Bounded Independence Graph.

b Recall that the Unit Disk Graph is defined for the Euclidean space and two nodes are connected if and only if their distance is at most c ($c = 1$ after normalization). Let $d(\cdot,\cdot)$ denote the normalized distance between two points. In our example, it holds that $d(v_2, v_5) = 1 > d(v_3, v_6) = 1/5$, but $\{v_2, v_5\} \in E$ and $\{v_3, v_6\} \notin E$. Therefore, this connectivity graph cannot be modeled as a UDG in the Euclidean plane.

c) The maximal QUDG ρ is given by the ratio of the shortest distance between two non-adjacent nodes divided by the largest edge. In G that is, $1/5 - \epsilon$ for some arbitrarily small $\epsilon > 0$.

d) The connectivity set E defines a UDG. A possible Euclidean representation is shown in Figure 3.

e) The graph defined by E is indeed a BIG, as we will see in the following. Observe that for nodes v_1, v_3, v_6 and v_7, the maximum independent set (MaxIS) in their 1-hop neighborhood is one. Nodes v_2, v_4 and v_5 have an independent set of size 2 in their 1-neighborhood. The MaxIS of the $\geq$ 2-neighborhood is 2—for all nodes. This clearly fulfills the BIG property, as there is essentially a constant upper bound of 2 on any MaxIS.

f) Let the distances of the nodes be as defined for the Euclidean plane. For this metric, the doubling dimension is at least $\log_2 4 = 2$: For instance, a ball of radius 2 centered at v_2 covers the entire clique v_1, v_2, v_3, v_4, but 4 balls are needed to cover the same set with balls of radius ≤ 1.

   Choosing a different metric for this connectivity graph can yield smaller dimensions. Consider the following metric: Nodes v_1, v_2, v_3 and v_4 have distances $1/4$, except for $d(v_3, v_4) = 1/8$ and $d(v_1, v_2) = 1/8$. Node v_5 has distance 1 to nodes v_2 and v_4. In addition, $d(v_5, v_6) = 1/2$, $d(v_5, v_7) = 1/2$, and $d(v_6, v_7) = 1/4$. All other distances are given by the shortest path metric, i.e., the shortest distance along the paths defined so far. Observe that these distances indeed define a metric, and convince yourself that the doubling dimension for this metric is 1: The nodes in each ball of radius r around a node $v \in V$ can always be covered with at most two balls of radius $r/2$.

g) Consider the following star graph, with a center node v. Node v has distance 1 to all other nodes $V \setminus v$, while nodes in $V \setminus v$ have distances 2 to each other. Clearly, this metric fulfills the non-negativity, symmetry and triangle inequality constraints. Moreover, a ball of radius

1 around v covers all other nodes. However, for radius 1/2, $|V|$ balls are required, yielding a *logarithmic* doubling dimension.

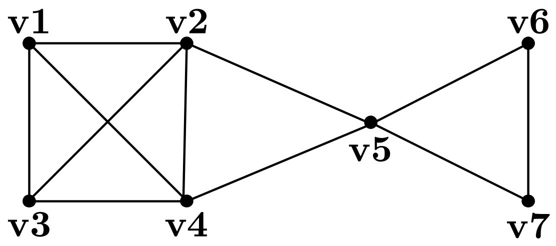

Figure 3: Example placement of nodes fulfilling UDG property and connectivity defined in E.