Exercise 12

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1 Vertex Coloring using All-to-All Communication

Consider an undirected graph $G = (V, E)$ with $n = |V|$ nodes and maximum degree $\Delta = \Omega(\log^3 n)$. Devise a randomized algorithm that with high probability computes a proper coloring of G with $O(\Delta)$ colors in $O(1)$ rounds of all-to-all communication, where in each round each node can send $O(\log n)$ bits to all other nodes.

Hint: *Think about randomly partitinoing nodes into several parts, and coloring each part separately, all in parallel.*

2 Edge Coloring

Consider an undirected graph $G = (V, E)$ with $n = |V|$ nodes and maximum degree Δ. Each edge $e \in E$ can be colored using several colors, that is, can be assigned a set $C(e) \subseteq \{1, \ldots, q\}$ of colors from a palette of size q. A color $c \in C(e)$ is *good* for the edge $e \in E$ if for all neighboring edges $f \in E$ we have $c \notin C(f)$. Suppose that we have access to a *checker* that informs the endpoints u and v of an edge $e = \{u, v\} \in E$ whether the colors in $C(e)$ are good.

(2a) Devise a randomized algorithm that invokes the checker once and with high probability finds a good color for every edge, using a total of $q = O(\Delta \log n)$ colors.

(2b) Devise a randomized algorithm that invokes the checker twice and with high probability finds a good color for every edge, using a total of $q = O(\Delta \log \log n)$ colors.

(2c) Explain how the above algorithms in (2a) and (2b), respectively, imply algorithms for the routing problem where the maximum number of messages starting from or destined to one node is at most $O(n/\log n)$ and $O(n/\log \log n)$, respectively, in the all-to-all communication setting, where in each round each node can send $O(\log n)$ bits to all other nodes.