Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

## Computer Engineering II

Solution to Exercise Sheet Chapter 13

## 1 Quiz Questions

a) No: The (supposed) security depends on not knowing the shift $x$. If CAESAR is applied twice, you just chose another shift (and in the worst case, cancel out the encryption).
b) No. E.g., $2 * 3 * 5-1=29$, which is prime. But $30 * 29-1=869=11 * 79$. Even the first part is not correct, e.g.: $2 * 3 * 5 * 7=210$ and $210-1=11 * 19$.
c) Yes. An attacker could just flip the bit of the message.
d) No. The attacker could just hash the modified message as well.

## 2 Secret Sharing

a) example execution: Let $a_{1}=3$ and $s=2$, with 2 neighbors. Thus, $f(x)=2+3 x$. We distribute, e.g., $(2,8)$ and $(3,11)$. With both pairs, $s=2$ can be recovered.
b) Without obtaining $t$ pairs, $k$ can take any value, i.e., $t-1$ pairs reveal no information on $k$.

## 3 The One Time Pad

a) If you apply the same one time pad twice, it cancels out, leaving you with the original message.
b) Essentially, you created a new one time pad. If both are truly random, then this method is not more secure, but also not less, it is the same.
c) The beauty of the one time pad is that it transforms the message into a random message. As thus, any string of length $k$ could be the original message - you still know nothing except for the length of the message.
d) Let $k$ be the OTP. $c_{1} \oplus c_{2}=m_{1} \oplus k \oplus m_{2} \oplus k=m_{1} \oplus m_{2}$, i.e., the one time pad cancels out. You don't have it decrypted yet, but it is a lot more information than just a random string.
e) You can get, e.g., $m_{3} \oplus m_{4}$, using similar techniques as above. I.e., $c_{3} \oplus c_{2}=m_{3} \oplus k$ and $c_{4} \oplus c_{3}=m_{4} \oplus k$, leading to $c_{4} \oplus c_{2}=m_{4} \oplus m_{3}$.

## 4 Diffie-Hellman Key Exchange

a) The primitive roots are 3 and 5 .
b) Alice sends $3^{4}=81=4 \bmod 7$ and Bob sends $3^{2}=9=2 \bmod 7$. As thus, they agree on $\left(3^{4}\right)^{2}=4^{2}=16=2 \bmod 7\left(\right.$ or $\left.\left(3^{2}\right)^{4}=2^{4}=16=2 \bmod 7\right)$.
c) (individual solutions)
d) Alice picked $k_{A}=3$, Bob picks $k_{B}=2$. Alice sends $3^{3}=27=2 \bmod 5$ to Bob and Bob sends $3^{2}=9=4 \bmod 5$ to Alice. As thus, they agree on $\left(3^{3}\right)^{2}=2^{2}=4 \bmod 5($ or $\left.\left(3^{2}\right)^{3}=4^{3}=64=4 \bmod 5\right)$.

## 5 Message Authentification

a) E.g., use sequence numbers.
b) The answer is no to both: Take any $m$ and $m^{\prime}=m+p$, then $h(m)=h\left(m^{\prime}\right)$. Similarly, given any $1 \leq m \leq p-1, h(m)=m$.
c) Use a large prime $p$ with a primitive root $g$. With $m$ being the message, let the hash be $h(m)=g^{m} \bmod p$. Now, finding an $x$ s.t. $h(x)=h(m)$ is the desired hash is equivalent to solving the discrete logarithm problem.

