Principles of Distributed Computing 04/26, 2017

Exercise 9

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Network Decompositions

Exercise 1: Explain how given a $(\mathcal{C}, \mathcal{D})$ network decomposition of graph G, a maximal independent set can be computed in $O(CD)$ rounds.

Exercise 2: We here see that the $(O(\log n), O(\log n))$ network decomposition that we discussed in the class has the nearly best possible parameters. In particular, it is known that there are n-node graphs that have girth[1] $\Omega(\log n/\log\log n)$ and chromatic number $\Omega(\log n)$[AS04, Erd59]. Use this fact to argue that on these graphs, an $(o(\log n), o(\log n/\log\log n))$ network decomposition does not exist.

Exercise 3: Given an n-node undirected graph $G = (V, E)$, we define a $d(n)$-*diameter ordering* of G to be a one-to-one labeling $f : V \to \{1, 2, \ldots, n\}$ of vertices such that for any path $P = v_1, v_2, \ldots, v_p$ on which the labels $f(v_i)$ are monotonically increasing, any two nodes $v_i, v_j \in P$ have $dist_G(v_i, v_j) \le d(n)$.

Use the existence of $(O(\log n), O(\log n))$ network decompositions, proved in the class, to argue that each n-node graph has an $O(\log^2 n)$-diameter ordering.

References

[AS04] Noga Alon and Joel H Spencer. *The probabilistic method.* John Wiley & Sons, 2004.

[Erd59] Paul Erdős. Graph theory and probability. *Canada J. Math*, 11:34G38, 1959.

[1] Recall that the girth of a graph is the length of its shortest cycle.