Exercise 6

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1 Regularized Luby's MIS Algorithm

Consider a regularized variant of Luby's MIS algorithm, as follows: The algorithm consists of $\log \Delta + 1$ phases, each made of $200 \log n$ consecutive steps. Here Δ denotes the maximum degree in the graph. In each step of the i^{th} phase, each remaining node is marked with probability $\frac{2^i}{10\Delta}$. Different nodes are marked independently. Then marked nodes who do not have any marked neighbor are added to the MIS, and removed from graph along with their neighbors. If at any time a node v becomes isolated, thus none of its neighbors remain, then v is also added to the MIS and gets removed from the graph.

(a) Argue that the set of vertices added to the MIS is always an *independent set*.

(b) Prove that with high probability, by the end of the i^{th} phase, in the remaining graph each node has degree at most $\frac{\Delta}{2^i}$.

(c) Conclude that the set of vertices added to the MIS is a *maximal* independent set, with high probability.

2 Randomized Coloring Algorithm

Consider the following simple randomized $\Delta + 1$ coloring algorithm: Per step, each node selects one of the colors not already *taken away* by its neighbors, at random. Then, if v selected a color and none of its neighbors selected the same color in that step, v gets colored with this color and takes this color away permanently. That is, none of the neighbors of v will select this color in any of the future steps.

(a) Prove that in the first step, each node has at least a constant probability of being colored.

(b*) Prove that per step each remaining node has at least a constant probability of being colored.

(b') If item (b) turns out to be complex, you may assume that we use $\lceil 1.02\Delta \rceil$ colors, instead of $\Delta + 1$. Prove that per step each remaining node has at least a constant probability of being colored.

(c) Conclude that within $200 \log n$ steps all nodes are colored, with high probability.