Concurrent Data Structures

Chapter 8



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Overview

- Concurrent Linked List
 - Fine-grained synchronization
 - Optimistic synchronization
 - Lazy synchronization
 - Lock-free synchronization
- Hashing
 - Fine-grained locking
 - Recursive split ordering

Handling Multiple Threads

- Adding threads should not lower the throughput
 - Contention effects can mostly be fixed by queue locks
- Adding threads should increase throughput
 - Not possible if the code is inherently sequential
 - Surprising things are parallelizable!
- How can we guarantee consistency if there are many threads?

Coarse-Grained Synchronization

- Each method locks the object
 - Avoid contention using queue locks
 - Mostly easy to reason about
 - This is the standard Java model (synchronized blocks and methods)
- Problem: Sequential bottleneck
 - Threads "stand in line"
 - Adding more threads does not improve throughput
 - We even struggle to keep it from getting worse...
- So why do we even use a multiprocessor?
 - Well, some applications are inherently parallel...
 - We focus on exploiting non-trivial parallelism

Exploiting Parallelism

- We will now talk about four "patterns"
 - Bag of tricks ...
 - Methods that work more than once ...
- The goal of these patterns are
 - Allow concurrent access
 - If there are more threads, the throughput increases!

Pattern #1: Fine-Grained Synchronization

- Instead of using a single lock split the concurrent object into independently-synchronized components
- Methods conflict when they access
 - The same component
 - At the same time

Pattern #2: Optimistic Synchronization

- Assume that nobody else wants to access your part of the concurrent object
- Search for the specific part that you want to lock without locking any other part on the way
- If you find it, try to lock it and perform your operations
 - If you don't get the lock, start over!
- Advantage
 - Usually cheaper than always assuming that there may be a conflict due to a concurrent access

Pattern #3: Lazy Synchronization

- Postpone hard work!
- Removing components is tricky
 - Either remove the object physically
 - Or logically: Only mark component to be deleted

Pattern #4: Lock-Free Synchronization

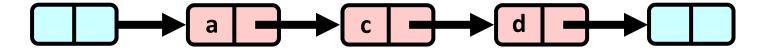
- Don't use locks at all!
 - Use compareAndSet() & other RMW operations!
- Advantages
 - No scheduler assumptions/support
- Disadvantages
 - Complex
 - Sometimes high overhead

Illustration of Patterns

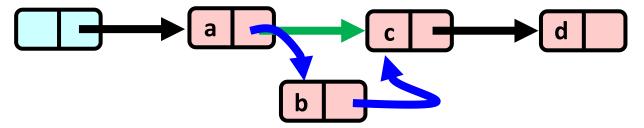
- In the following, we will illustrate these patterns using a list-based set
 - Common application
 - Building block for other apps
- A set is a collection of items
 - No duplicates
- The operations that we want to allow on the set are
 - add(x) puts x into the set
 - remove(x) takes x out of the set
 - contains(x) tests if x is in the set

The List-Based Set

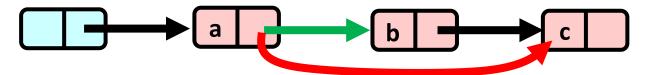
 We assume that there are sentinel nodes at the beginning (head) and end (tail) of the linked list



Add node b:

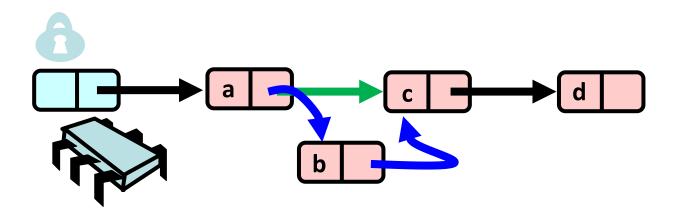


Remove node b:



Coarse-Grained Locking

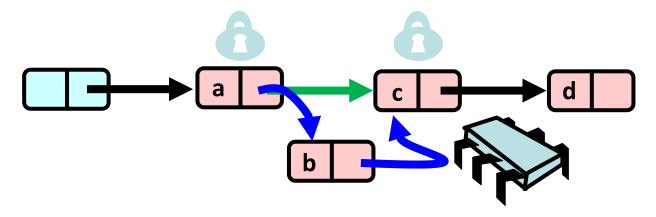
- A simple solution is to lock the entire list for each operation
 - E.g., by locking the head



- Simple and clearly correct!
- Works poorly with contention...

Fine-Grained Locking

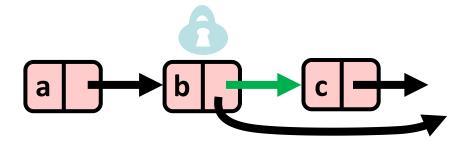
- Split object (list) into pieces (nodes)
 - Each piece (each node in the list) has its own lock
 - Methods that work on disjoint pieces need not exclude each other



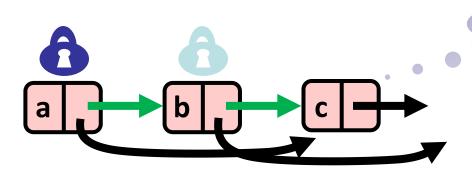
- Hand-over-hand locking: Use two locks when traversing the list
 - Why two locks?

Problem with One Lock

- Assume that we want to delete node c
- We lock node b and set its next pointer to the node after c



 Another thread may concurrently delete node b by setting the next pointer from node a to node c

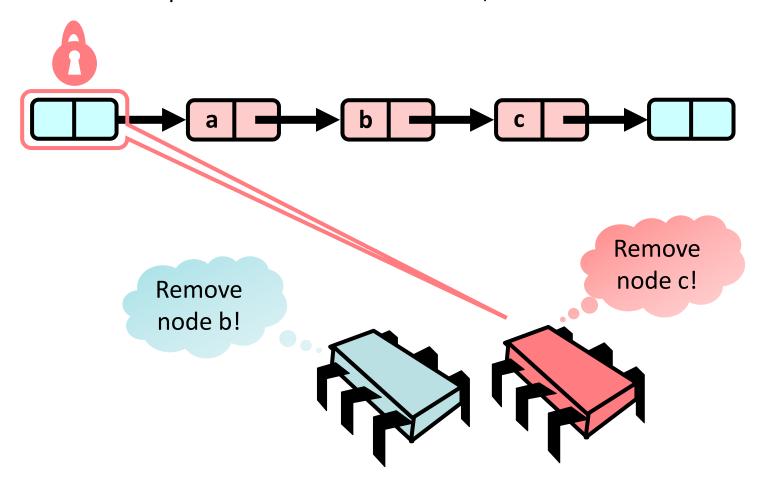


Hooray, I'm not deleted!

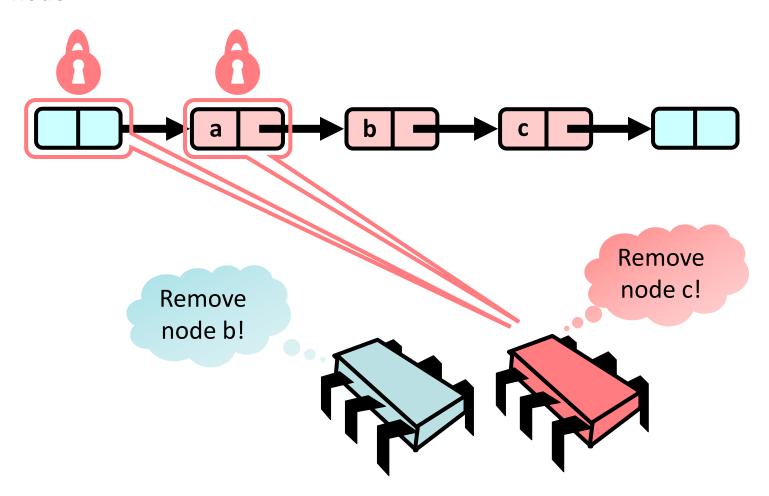
Insight

- If a node is locked, no one can delete the node's *successor*
- If a thread locks
 - the node to be deleted
 - and also its predecessor
- then it works!
- That's why we (have to) use two locks!

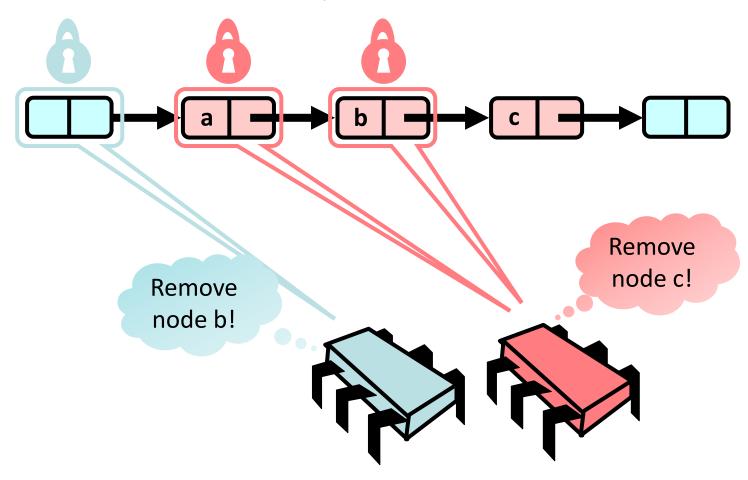
- Assume that two threads want to remove the nodes b and c
- One thread acquires the lock to the sentinel, the other has to wait



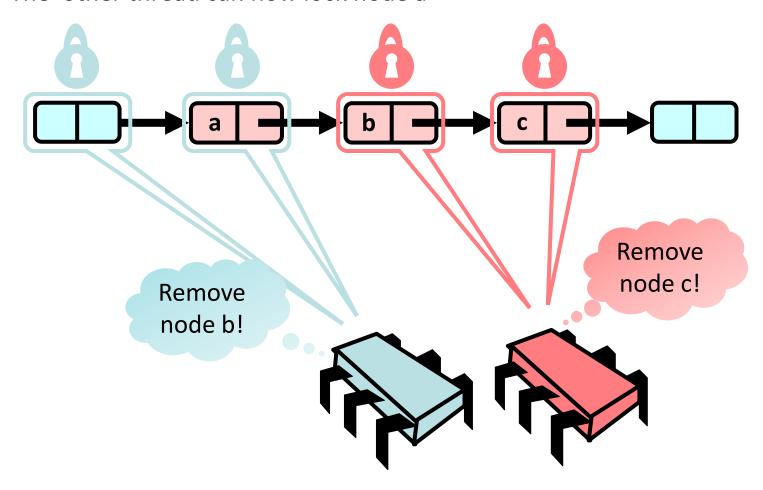
 The same thread that acquired the sentinel lock can then lock the next node



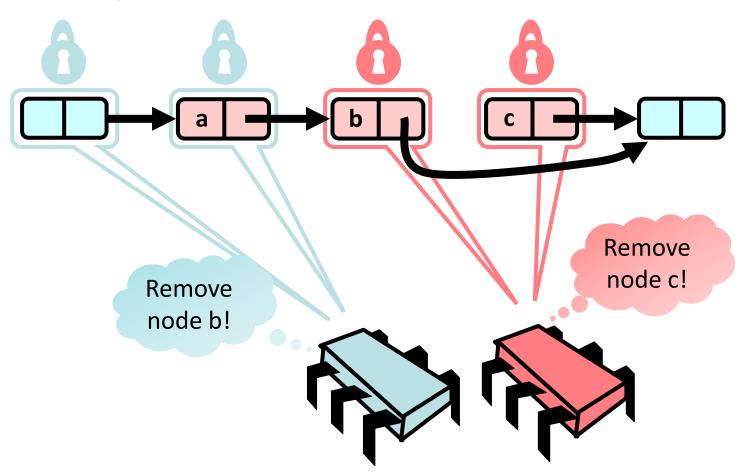
- Before locking node b, the sentinel lock is released
- The other thread can now acquire the sentinel lock



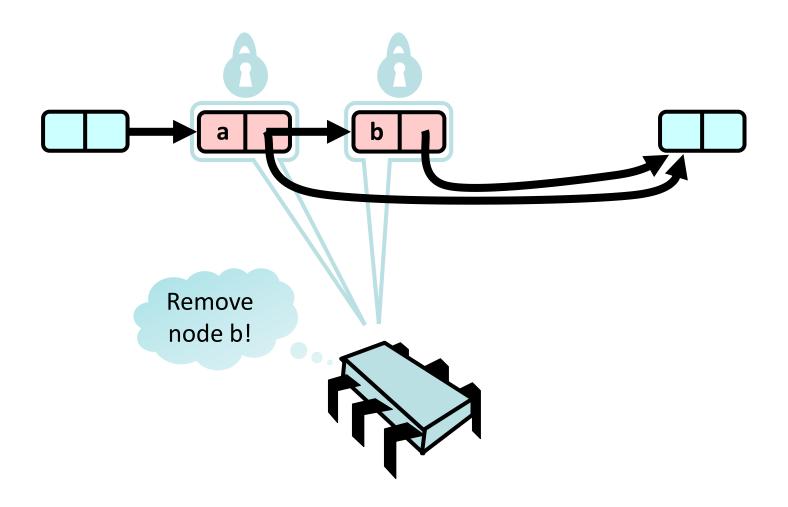
- Before locking node c, the lock of node a is released
- The other thread can now lock node a



- Node c can now be removed
- Afterwards, the two locks are released



The other thread can now lock node b and remove it



List Node

```
public class Node {
    public T item;
    public int key;
    public Node next;
}

    Reference to next node
```

Remove Method

```
public boolean remove(Item item) {
  int key = item.hashCode();
  Node pred, curr;
                                Start at the head and lock it
  trv
    pred = this.head;
    pred.lock();
                                 Lock the current node
    curr = pred.next;
    curr.lock();
                                Traverse the list and
                                  remove the item
                                                     On the
    finally {
                                                    next slide!
       curr.unlock();
                                 Make sure that the
       pred.unlock();
                                 locks are released
```

Remove Method

```
while (curr.key <= key) {
    if (item == curr.item) {
        pred.next = curr.next;
        return true;
    }

    pred.unlock();
    pred = curr;
    curr = curr.next;
    curr = curr.next;
    curr lock();
}

return false;

Return false if the element is not present</pre>
```

Why does this work?

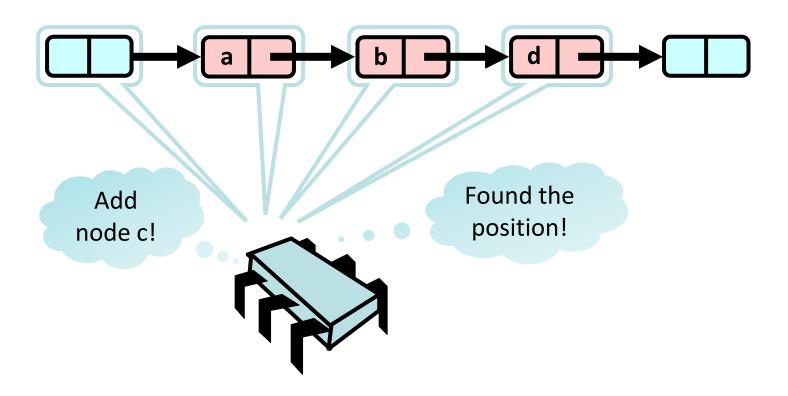
- To remove node e
 - Node e must be locked
 - Node e's predecessor must be locked
- Therefore, if you lock a node
 - It can't be removed
 - And neither can its successor
- To add node e
 - Must lock predecessor
 - Must lock successor
- Neither can be deleted
 - Is the successor lock actually required?

Drawbacks

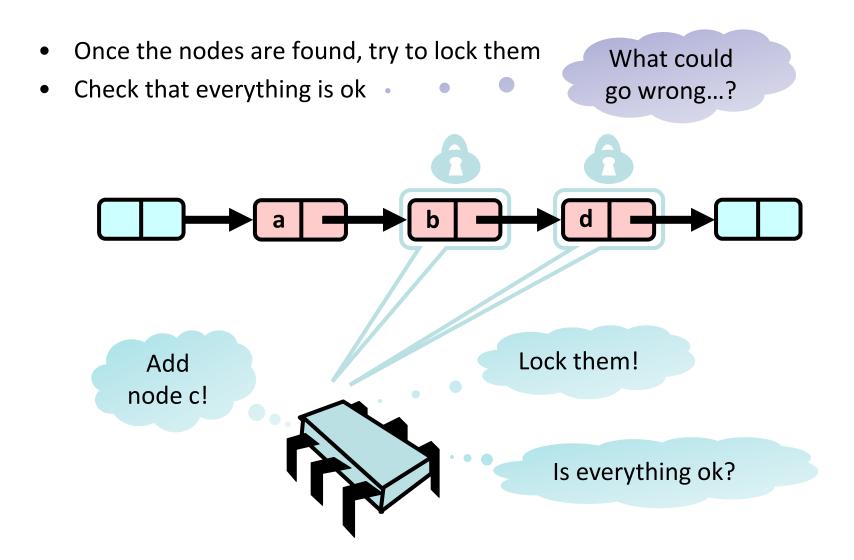
- Hand-over-hand locking is sometimes better than coarse-grained locking
 - Threads can traverse in parallel
 - Sometimes, it's worse!
- However, it's certainly not ideal
 - Inefficient because many locks must be acquired and released
- How can we do better?

Optimistic Synchronization

Traverse the list without locking!

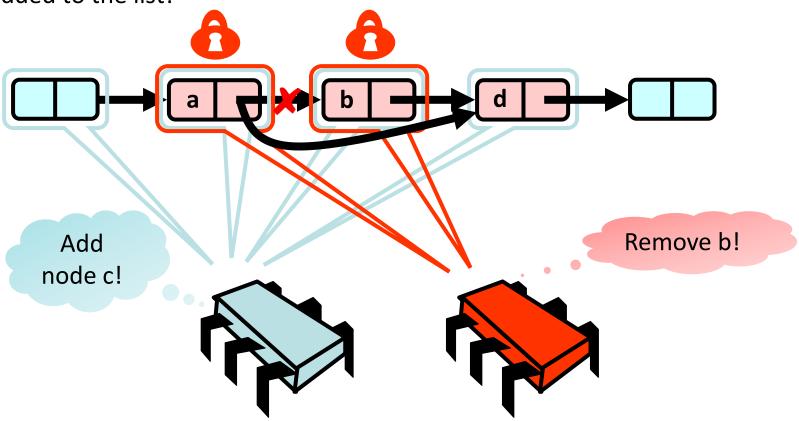


Optimistic Synchronization: Traverse without Locking



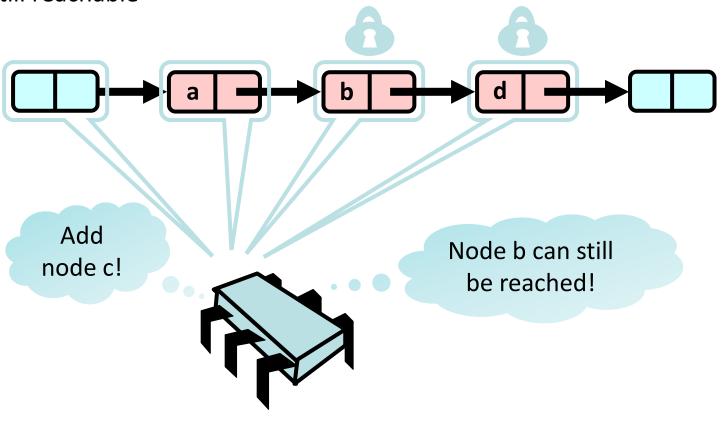
Optimistic Synchronization: What Could Go Wrong?

 Another thread may lock nodes a and b and remove b before node c is added → If the pointer from node b is set to node c, then node c is not added to the list!



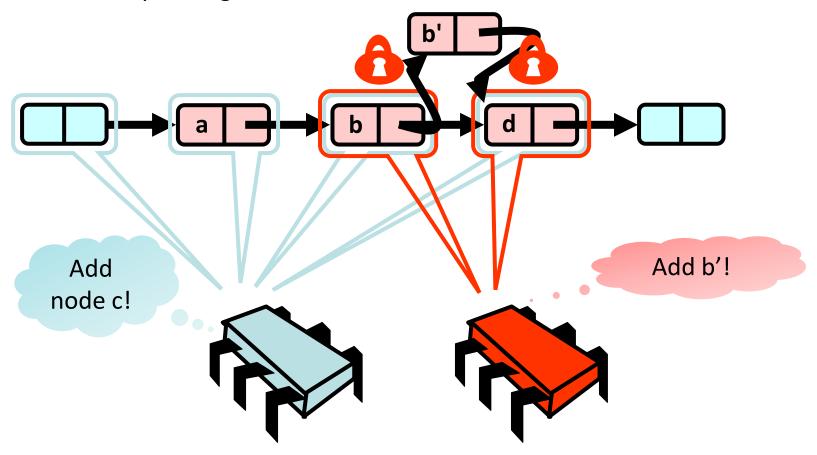
Optimistic Synchronization: Validation #1

- How can this be fixed?
- After locking node b and node d, traverse the list again to verify that b is still reachable



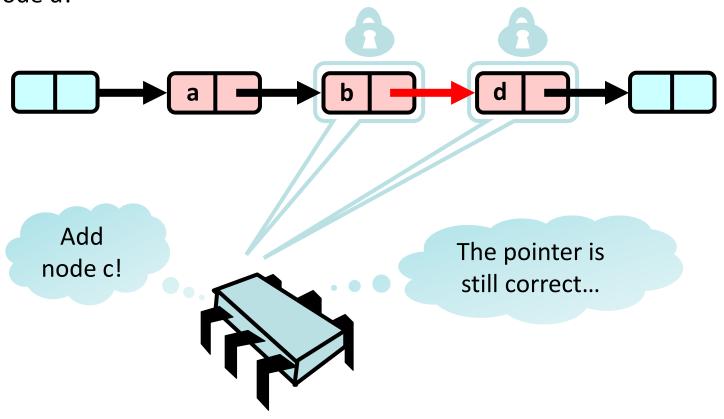
Optimistic Synchronization: What Else Could Go Wrong?

 Another thread may lock nodes b and d and add a node b' before node c is added → By adding node c, the addition of node b' is undone!



Optimistic Synchronization: Validation #2

- How can this be fixed?
- After locking node b and node d, also check that node b still points to node d!



Optimistic Synchronization: Validation

```
private boolean validate(Node pred, Node curr) {
  Node node = head;
  while (node.key <= pred.key) {
    if (node == pred)
        return pred.next == curr;
        node = node.next;
  }
  return false;
}</pre>

Predecessor not reachable
```

Optimistic Synchronization: Remove

```
private boolean remove(Item item) {
  int key = item.hashCode();
                                        Retry on synchronization
 while (true) { 🗗
                                               conflict
    Node pred = this.head;
    Node curr = pred.next;
    while (curr.key <= key) {</pre>
      if (item == curr.item)
                                       Stop if we find the item
         break;
      pred = curr;
      curr = curr.next;
```

Optimistic Synchronization: Remove

```
trv
                                           Lock both nodes
 pred.lock(); curr.lock();
  if (validate(pred,curr))
                                          Check for
    if (curr.item == item)
                                    synchronization conflicts
      pred.next = curr.next;
      return true;
                                     Remove node if
    } else {
                                       target found
      return false;
  finally {
  pred.unlock();
  curr.unlock();
                        Always unlock the nodes
```

Optimistic Synchronization

- Why is this correct?
 - If nodes b and c are both locked, node b still accessible, and node c still the successor of node b, then neither b nor c will be deleted by another thread
 - This means that it's ok to delete node c!
- Why is it good to use optimistic synchronization?
 - Limited hot-spots: no contention on traversals
 - Fewer lock acquisitions and releases
- When is it good to use optimistic synchronization?
 - When the cost of scanning twice without locks is less than the cost of scanning once with locks
- Can we do better?
 - It would be better to traverse the list only once...

Lazy Synchronization

- Key insight
 - Removing nodes causes trouble
 - Do it "lazily"
- How can we remove nodes "lazily"?
 - First perform a logical delete: Mark current node as removed (new!)



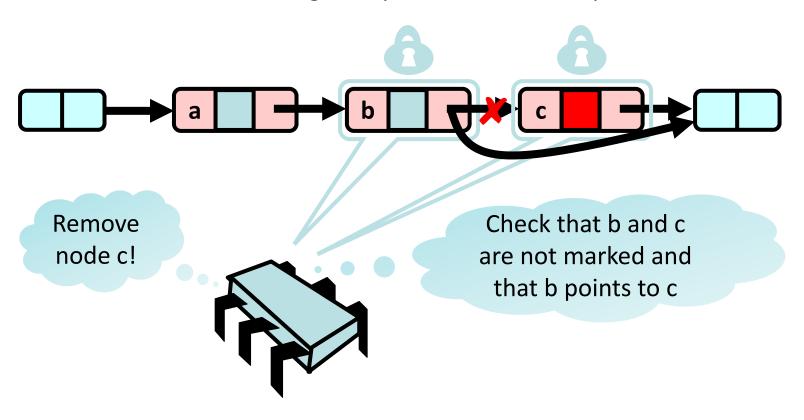
Then perform a physical delete: Redirect predecessor's next (as before)

Lazy Synchronization

- All Methods
 - Scan through locked and marked nodes
 - Removing a node doesn't slow down other method calls...
- Note that we must still lock pred and curr nodes!
- How does validation work?
 - Check that neither pred nor curr are marked
 - Check that pred points to curr

Lazy Synchronization

- Traverse the list and then try to lock the two nodes
- Validate!
- Then, mark node c and change the predecessor's next pointer



Lazy Synchronization: Validation

```
private boolean validate(Node pred, Node curr) {
    return !pred.marked && !curr.marked &&
    pred.next == curr;
}
    Nodes are not
    logically removed
```

Predecessor still points to current

Lazy Synchronization: Remove

```
public boolean remove(Item item) {
  int key = item.hashCode();
  while (true) {
    Node pred = this.head;
    Node curr = pred.next;
    while (curr.key <= key) {</pre>
      if (item == curr.item)
        break;
      pred = curr;
      curr = curr.next;
```

This is the same as before!

Lazy Synchronization: Remove

```
try {
  pred.lock(); curr.lock();
 if (validate(pred,curr))
                                          Check for
    if (curr.item == item) {
                                    synchronization conflicts
      curr.marked = true;
      pred.next = curr.next;
      return true:
                                     If the target is found,
    } else {
                                      mark the node and
      return false;
                                          remove it
} finally {
  pred.unlock();
  curr.unlock();
```

Lazy Synchronization: Contains

```
public boolean contains(Item item) {
  int key = item.hashCode();
  Node curr = this.head;
  while (curr.key < key) {
    curr = curr.next;
  }
    return curr.item == item && !curr.marked;
}</pre>
Traverse without locking
(nodes may have been
removed)
```

Is the element present and not marked?

Evaluation

Good

- The list is traversed only once without locking
- Note that contains() doesn't lock at all!
- This is nice because typically contains() is called much more often than add() or remove()
- Uncontended calls don't re-traverse

Bad

- Contended add() and remove() calls do re-traverse
- Traffic jam if one thread delays

Traffic jam?

- If one thread gets the lock and experiences a cache miss/page fault, every other thread that needs the lock is stuck!
- We need to trust the scheduler....

Lock-Free Data Structures

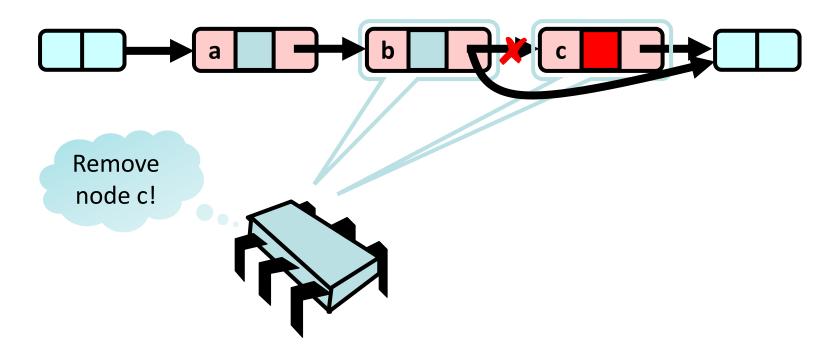
 If we want to guarantee that some thread will eventually complete a method call, even if other threads may halt at malicious times, then the implementation cannot use locks!



- Next logical step: Eliminate locking entirely!
- Obviously, we must use some sort of RMW method
- Let's use CompareAndSet() (CAS)!

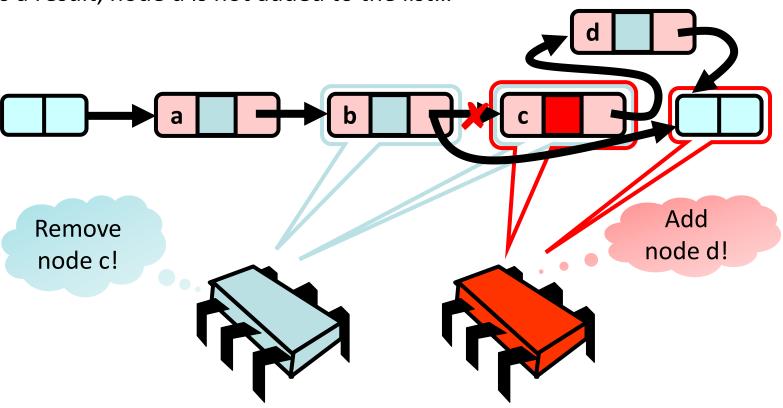
Remove Using CAS

- First, remove the node logically (i.e., mark it)
- Then, use CAS to change the next pointer
- Does this work...?



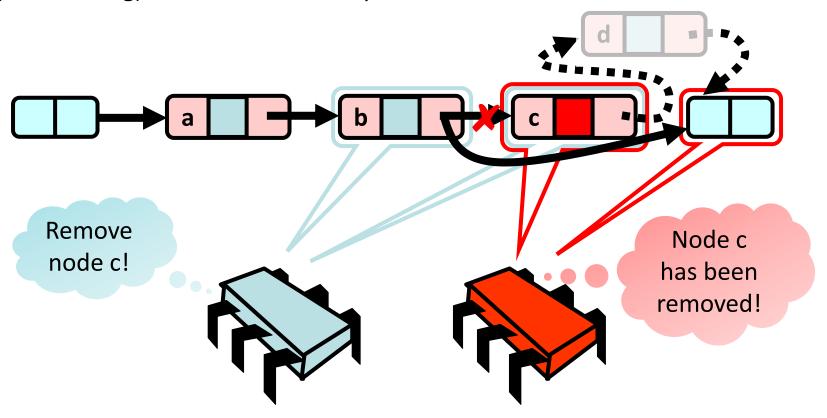
Remove Using CAS: Problem

- Unfortunately, this doesn't work!
- Another node d may be added before node c is physically removed
- As a result, node d is not added to the list...



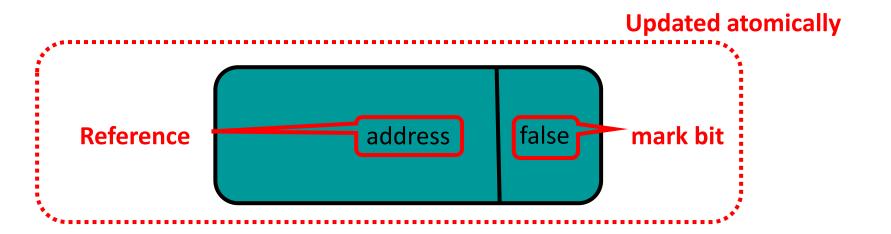
Solution

- Mark bit and next pointer are "CASed together"
- This atomic operation ensures that no node can cause a conflict by adding (or removing) a node at the same position in the list



Solution

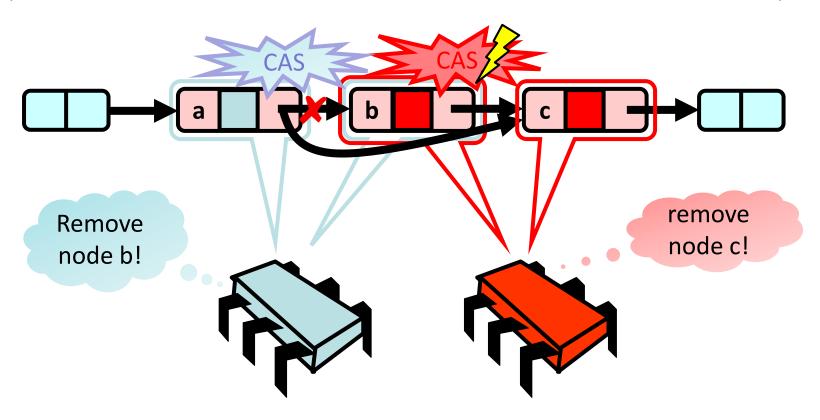
- Such an operation is called an atomic markable reference
 - Atomically update the mark bit and redirect the predecessor's next pointer
- In Java, there's an AtomicMarkableReference class
 - In the package Java.util.concurrent.atomic package



Changing State

Removing a Node

- If two threads want to delete the nodes b and c, both b and c are marked
- The CAS of the red thread fails because node b is marked!
- (If node b is not marked, then b is removed first and there is no conflict)



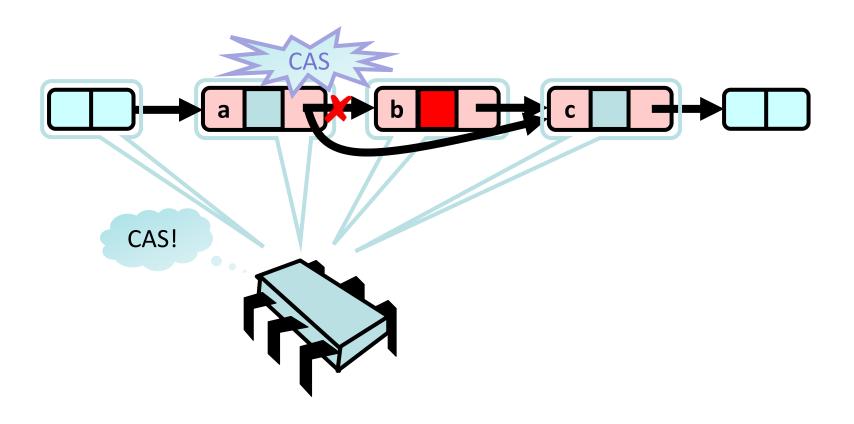
Traversing the List

• Question: What do you do when you find a "logically" deleted node in your path when you're traversing the list?



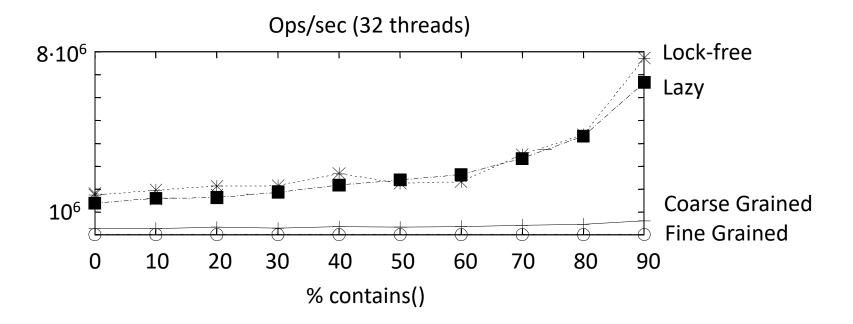
Lock-Free Traversal

• If a logically deleted node is encountered, CAS the predecessor's next field and proceed (repeat as needed)



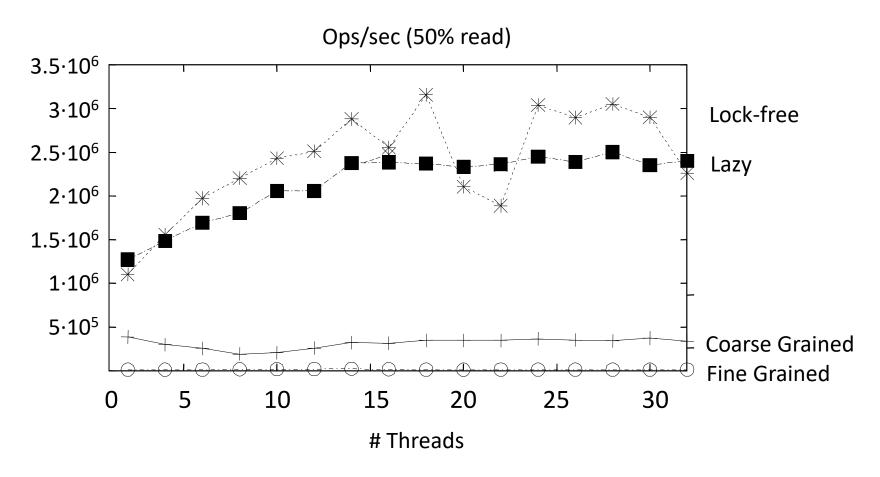
Performance

- The throughput of the presented techniques has been measured for a varying percentage of contains() method calls
 - Using a benchmark on a 16 node shared memory machine



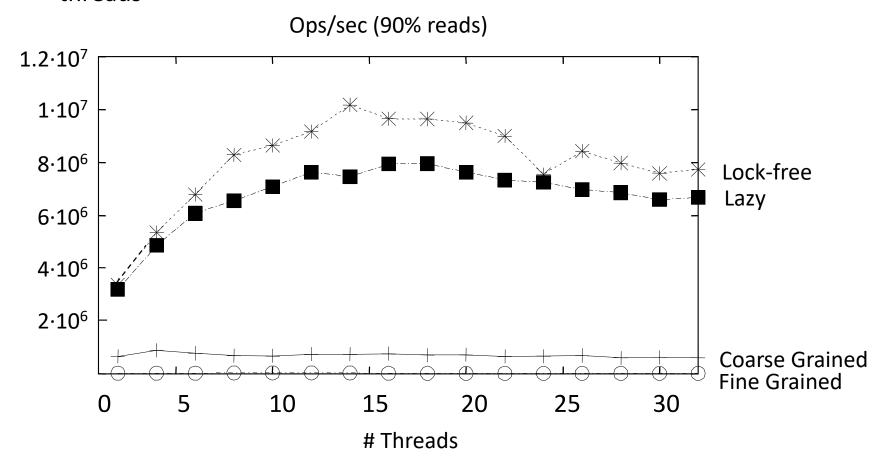
Low Ratio of contains()

• If the ratio of contains() is low, the lock-free linked list and the linked list with lazy synchronization perform well even if there are many threads



High Ratio of contains()

 If the ratio of contains() is high, again both the lock-free linked list and the linked list with lazy synchronization perform well even if there are many threads



"To Lock or Not to Lock"

- Locking vs. non-blocking: Extremist views on both sides
- It is nobler to compromise by combining locking and non-blocking techniques
 - Example: Linked list with lazy synchronization combines blocking add() and remove() and a non-blocking contains()
 - Blocking/non-blocking is a property of a method

Linear-Time Set Methods

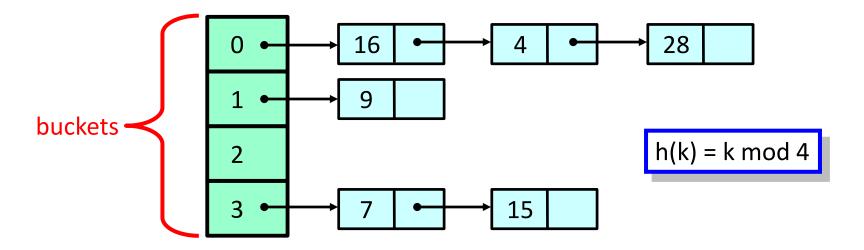
- We looked at a number of ways to make highly-concurrent list-based sets
 - Fine-grained locks
 - Optimistic synchronization
 - Lazy synchronization
 - Lock-free synchronization
- What's not so great?
 - add(), remove(), contains() take time linear in the set size
- We want constant-time methods! • How...?
 - At least on average...

Hashing

- A hash function maps the items to integers
 - h: items → integers
- Uniformly distributed
 - Different items "most likely" have different hash values
- In Java there is a hashCode() method

Sequential Hash Map

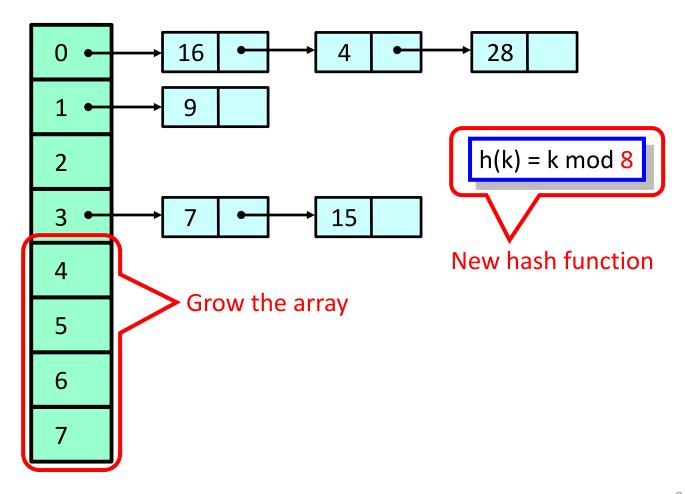
 The hash table is implemented as an array of buckets, each pointing to a list of items



- Problem: If many items are added, the lists get long → Inefficient lookups!
- Solution: Resize!

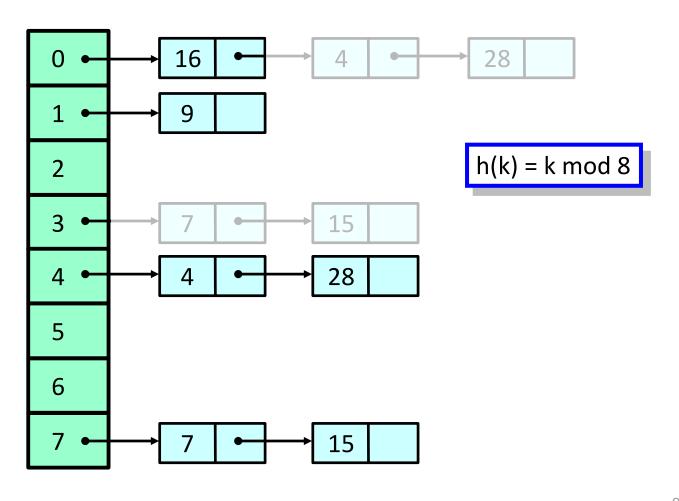
Resizing

The array size is doubled and the hash function adjusted



Resizing

Some items have to be moved to different buckets!



Hash Sets

- A hash set implements a set object
 - Collection of items, no duplicates
 - add(), remove(), contains() methods
- More coding ahead!



Simple Hash Set

```
public class SimpleHashSet {
 protected LockFreeList[] table;
                                        Array of lock-free lists
                                         Initial size
  public SimpleHashSet(int capacity)
    table = new LockFreeList[capacity];
    for (int i = 0; i < capacity; i++)
                                                Initialization
      table[i] = new LockFreeList();
  public boolean add(Object key) {
    int hash = key.hashCode() % table.length;
    return table[hash].add(key);
```

Use hash of object to pick a bucket and call bucket's add() method

Simple Hash Set: Evaluation

- We just saw a
 - Simple
 - Lock-free
 - Concurrent

hash-based set implementation

- But we don't know how to resize...
- Is Resizing really necessary?
 - Yes, since constant-time method calls require constant-length buckets and a table size proportional to the set size
 - As the set grows, we must be able to resize

Set Method Mix

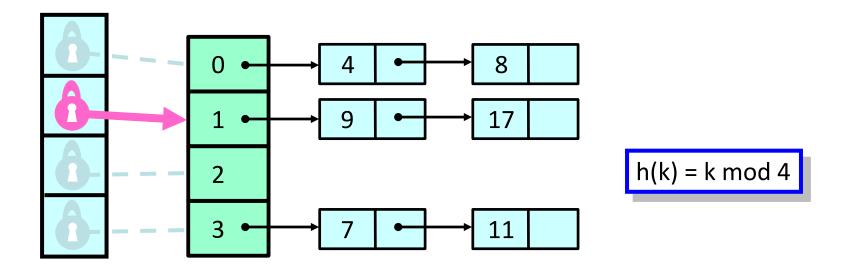
- Typical load
 - 90% contains()
 - 9% add ()
 - 1% remove()
- Growing is important, shrinking not so much
- When do we resize?
- There are many reasonable policies, e.g., pick a threshold on the number of items in a bucket
- Global threshold
 - When, e.g., ≥ ¼ buckets exceed this value
- Bucket threshold
 - When any bucket exceeds this value

Coarse-Grained Locking

- If there are concurrent accesses, how can we safely resize the array?
- As with the linked list, a straightforward solution is to use coarse-grained locking: lock the entire array!
- This is very simple and correct
- However, we again get a sequential bottleneck...
- How about fine-grained locking?

Fine-Grained Locking

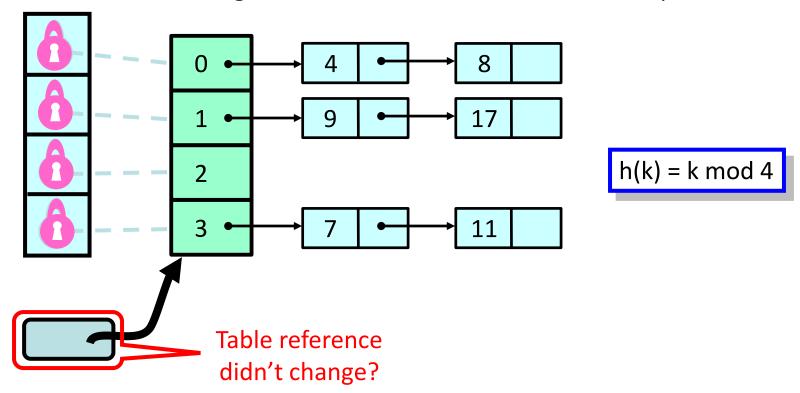
Each lock is associated with one bucket



After acquiring the lock of the list, insert the item in the list!

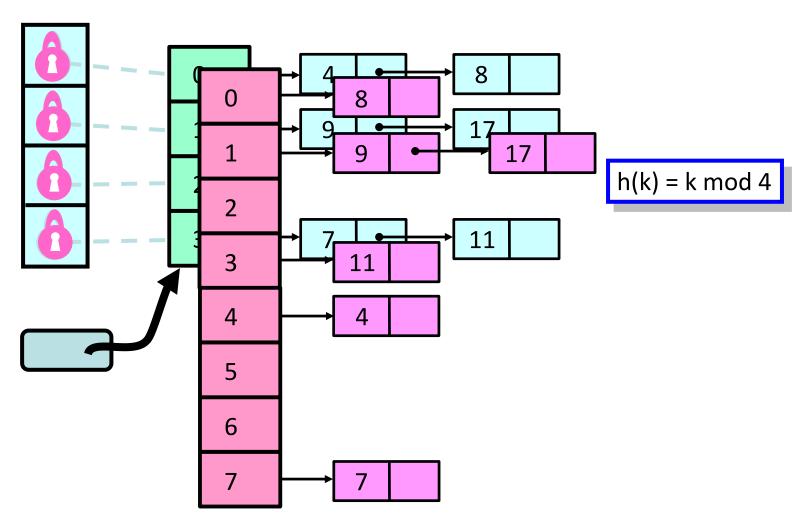
Fine-Grained Locking: Resizing

 Acquire all locks in ascending order and make sure that the table reference didn't change between resize decision and lock acquisition!



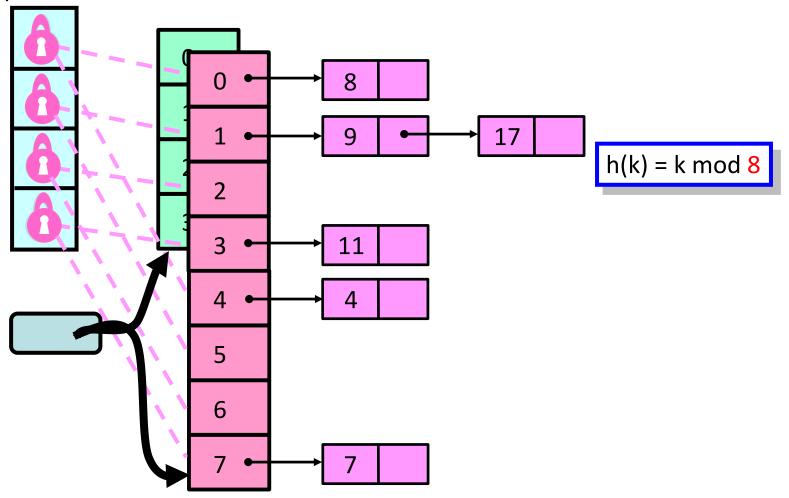
Fine-Grained Locking: Resizing

Allocate a new table and copy all elements



Fine-Grained Locking: Resizing

- Stripe the locks: Each lock is now associated with two buckets
- Update the hash function and the table reference



Observations

- We grow the table, but we don't increase the number of locks
 - Resizing the lock array is possible, but tricky...
- We use sequential lists (coarse-grained locking)
 - No lock-free list
 - If we're locking anyway, why pay?

Fine-Grained Hash Set

```
public class FGHashSet {
                                    Array of locks
 protected RangeLock[] lock;
                                    Array of buckets
 protected List[] table;
  public FGHashSet(int capacity) {
    table = new List[capacity];
    lock = new RangeLock[capacity];
    for (int i = 0; i < capacity; i++)
                                            Initially the same
      lock[i] = new RangeLock();
                                              number of locks
      table[i] = new LinkedList();
                                               and buckets
```

Fine-Grained Hash Set: Add Method

```
public boolean add(Object key) {
    int keyHash = key.hashCode() % lock.length;
    right lock
    synchronized (lock[keyHash]) {
        int tableHash = key.hashCode() % table.length;
        return table[tableHash].add(key);
    }
}

Call the add() method of
    the right bucket
```

Fine-Grained Hash Set: Resize Method

```
public void resize(int depth, List[] oldTable)
  synchronized (lock[depth]) {
                                                 Resize() calls
    if (oldTable == this.table)
                                               resize(0,this.table)
      int next = depth + 1;
                                          Acquire the next
       if (next < lock.length)</pre>
                                           lock and check
         resize(next, oldTable);
                                           that no one else
       else
                                             has resized
        sequentialResize();
                                  Recursively acquire
                                     the next lock
            Once the locks are
           acquired, do the work
```

Fine-Grained Locks: Evaluation

- We can resize the table, but not the locks
- It is debatable whether method calls are constant-time in presence of contention ...
- Insight: The contains() method does not modify any fields
 - Why should concurrent contains() calls conflict?

Read/Write Locks

Lock Safety Properties

- No thread may acquire the write lock
 - while any thread holds the write lock
 - or the read lock
- No thread may acquire the read lock
 - while any thread holds the write lock
- Concurrent read locks OK
- This satisfies the following safety properties
 - If readers > 0 then writer == false
 - If writer = true then readers == 0

Read/Write Lock: Liveness

- How do we guarantee liveness?
 - If there are lots of readers, the writers may be locked out!
- Solution: FIFO Read/Write lock
 - As soon as a writer requests a lock, no more readers are accepted
 - Current readers "drain" from lock and the writers acquire it eventually

Optimistic Synchronization

- What if the contains() method scans without locking...?
- If it finds the key
 - It is ok to return true!
 - Actually requires a proof...

We won't discuss this in this lecture

- What if it doesn't find the key?
 - It may be a victim of resizing...
 - Get a read lock and try again!
 - This makes sense if it is expected(?) that the key is there and resizes are rare.
 - Better: Check if the table size is the same before and after the method call!

Stop The World Resizing

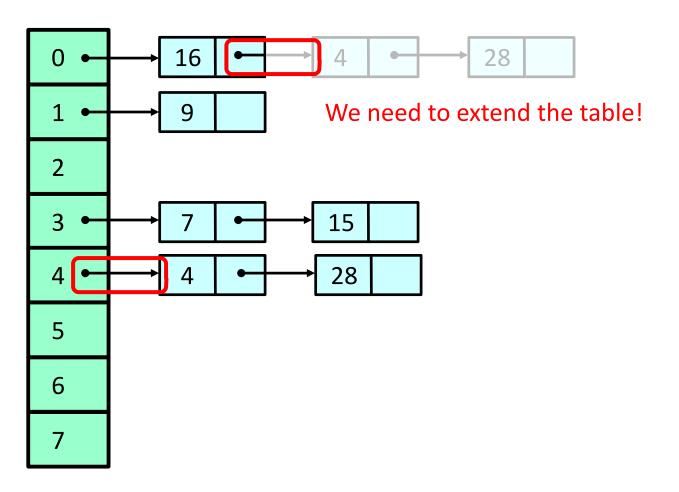
- The resizing we have seen up till now stops all concurrent operations
- Can we design a resize operation that will be incremental?
- We need to avoid locking the table...

How...?

We want a lock-free table with incremental resizing!

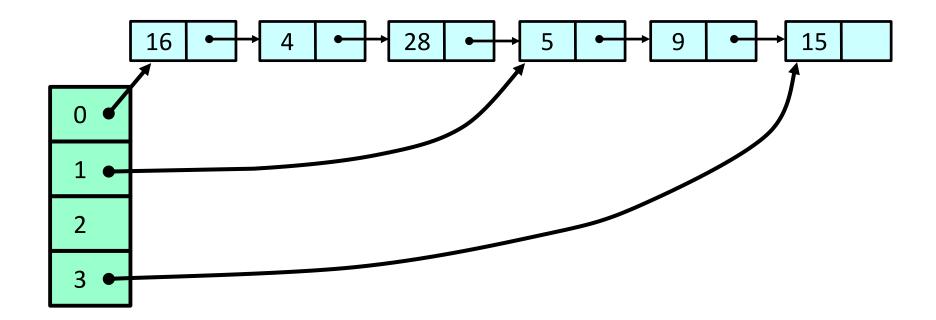
Lock-Free Resizing Problem

 In order to remove and then add even a single item, "single location CAS" is not enough...



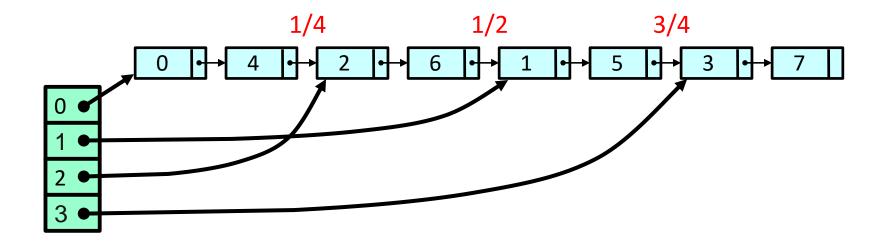
Idea: Don't Move the Items

- Move the buckets instead of the items!
- Keep all items in a single lock-free list
- Buckets become "shortcut pointers" into the list



Recursive Split Ordering

- Example: The items 0 to 7 need to be hashed into the table
- Recursively split the buckets in half:

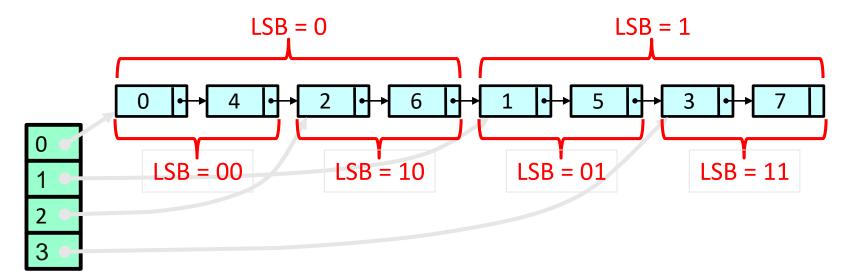


The list entries are sorted in an order that allows recursive splitting



Recursive Split Ordering

 Note that the least significant bit (LSB) is 0 in the first half and 1 in the other half! The second LSB determines the next pointers etc.

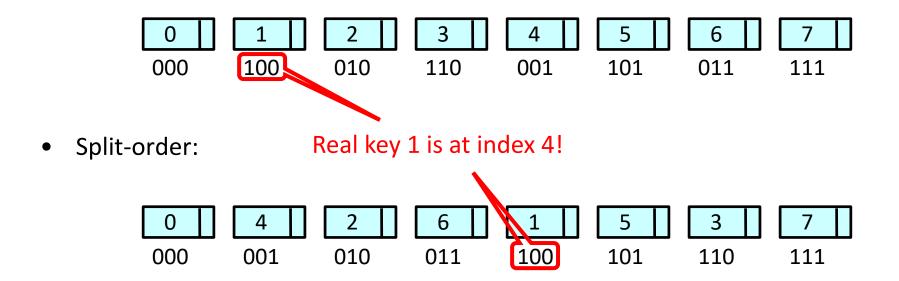


Split-Order

- If the table size is 2ⁱ:
 - Bucket b contains keys k = b mod 2ⁱ
 - The bucket index consists of the key's i least significant bits
- When the table splits:
 - Some keys stay (b = k mod 2^{i+1})
 - Some keys move (b+2 i = k mod 2 $^{i+1}$)
- Whether a key moves is determined by the (i+1)st bit
 - counting backwards

A Bit of Magic

- We need to map the real keys to the split-order
- Look at the reversed binary representation of the keys and the indices
- The real keys:



Just reverse the order of the key bits in order to get the index!

Split Ordered Hashing

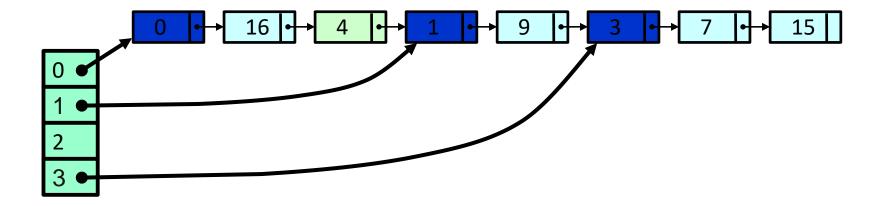
• After a resize, the new pointers are found by searching for the right index

Order according to reversed bits 000 001 010 011 100 101 110 111 0 + 4 + 2 + 6 + 1 + 5 + 3 + 7 2 pointers to some nodes!

 A problem remains: How can we remove a node by means of a CAS if two sources point to it?

Sentinel Nodes

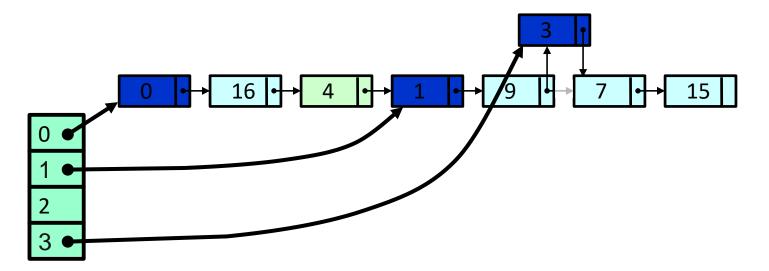
• Solution: Use a sentinel node for each bucket



- We want a sentinel key for i
 - before all keys that hash to bucket i
 - after all keys that hash to bucket (i-1)

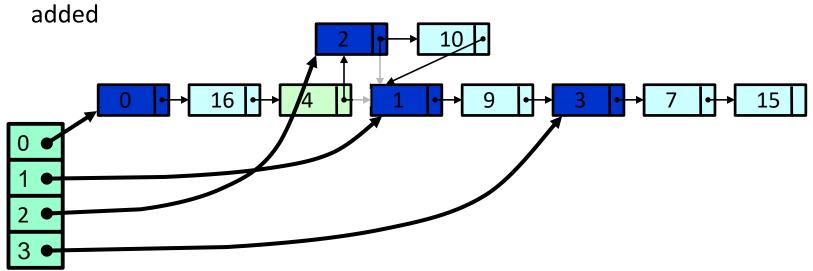
Initialization of Buckets

- We can now split a bucket in a lock-free manner using two CAS() calls
- Example: We need to initialize bucket 3 to split bucket 1!



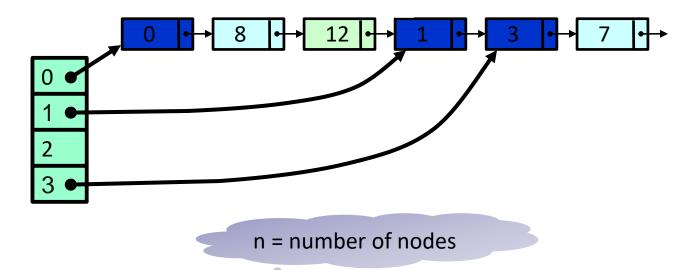
Adding Nodes

- Example: Node 10 is added
- First, bucket 2 (= 10 mod 4) must be initialized, then the new node is



Recursive Initialization

- It is possible that buckets must be initialized recursively
- Example: When node 7 is added, bucket 3 (= 7 mod 4) is initialized and then bucket 1 (= 3 mod 2) is also initialized



 Note that ≈ log n empty buckets may be initialized if one node is added, but the expected depth is constant!

Lock-Free List

Split-Ordered Set

```
public class SOSet{
                                          This is the lock-free
  protected LockFreeList[] table;
                                            list with minor
  protected AtomicInteger tableSize;
                                             modifications
  protected AtomicInteger setSize;
                                            Track how much of
  public SOSet(int capacity) {
                                           the table is used and
    table = new LockFreeList[capacity];
                                            the set size so that
    table[0] = new LockFreeList();
                                            we know when to
    tableSize = new AtomicInteger(1);
                                                 resize
    setSize = new AtomicInteger(0);
```

Initially use 1 bucket and the size is 0

Split-Ordered Set: Add

```
public boolean add(Object object) {
                                                Pick a bucket
  int hash = object.hashCode();
  int bucket = hash % tableSize.get();
                                                Non-sentinel
  int key = makeRegularKey(hash);
                                              split-ordered key
  LockFreeList list = getBucketList(bucket);
  if (!list.add(object,key))
                                                Get pointer to
    return false;
                                               bucket's sentinel,
                               Try to add with
  resizeCheck();
                                                 initializing if
                                reversed key
  return true,
                                                  necessary
                      Resize if
                     necessary
```

Recall: Resizing & Initializing Buckets

- Decision to Resize
 - Divide the set size by the total number of buckets
 - If the quotient exceeds a threshold, double the table size up to a fixed limit
- Initializing Buckets
 - Buckets are originally null
 - If you encounter a null bucket, initialize it
 - Go to bucket's parent (earlier nearby bucket) and recursively initialize if necessary
 - Constant expected work per bucket!

Split-Ordered Set: Initialize Bucket

```
public void initializeBucket(int bucket) {
    int parent = getParent(bucket);
    if (table[parent] == null)
        initializeBucket(parent);
    int key = makeSentinelKey(bucket);
    table[bucket] = new
        LockFreeList(table[parent], key);
}

    Prepare key for
        new sentinel
```

Insert sentinel if not present and return reference to rest of list

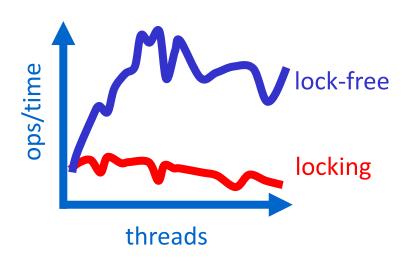
Correctness

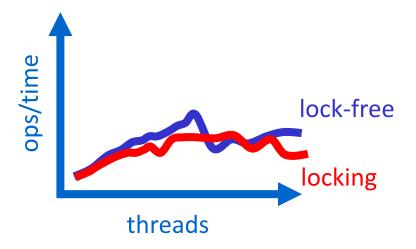
- Split-ordered set is a correct, linearizable, concurrent set implementation
- Constant-time operations!
 - It takes no more than O(1) items between two dummy nodes on average
 - Lazy initialization causes at most O(1) expected recursion depth in initializeBucket()

Empirical Evaluation

- Evaluation has been performed on a 30-processor Sun Enterprise 3000
- Lock-Free vs. fine-grained optimistic locking ("Lea")
- 10⁶ operations: 88% contains(), 10% add(), 2% remove()

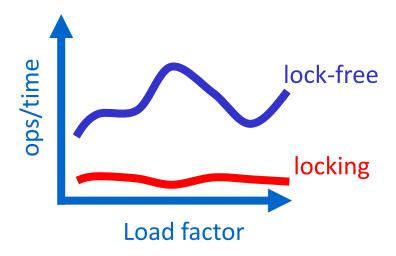
Low load: High load:



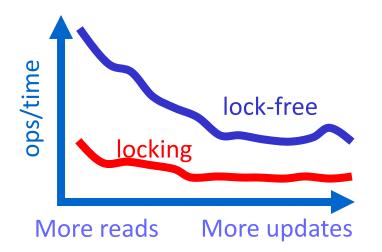


Empirical Evaluation

- Expected bucket length
 - The load factor is the capacity of the individual buckets



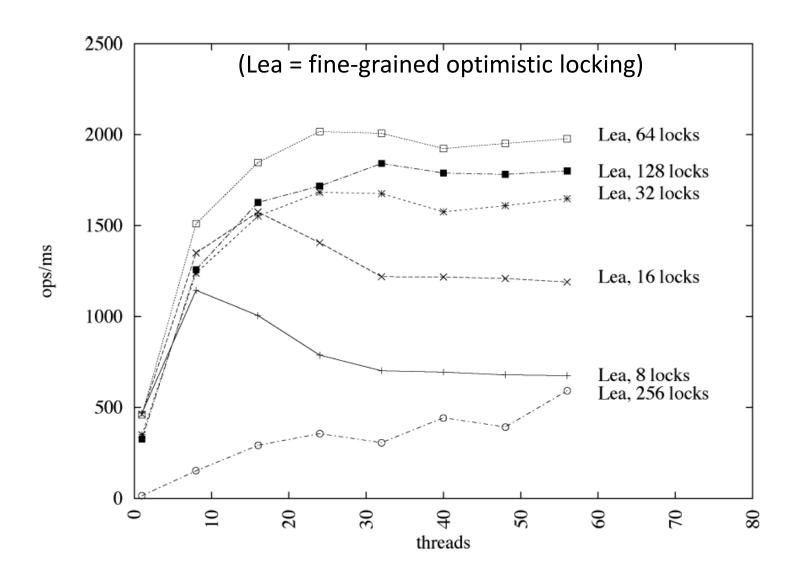
- Varying The Mix
 - Increasing the number of updates



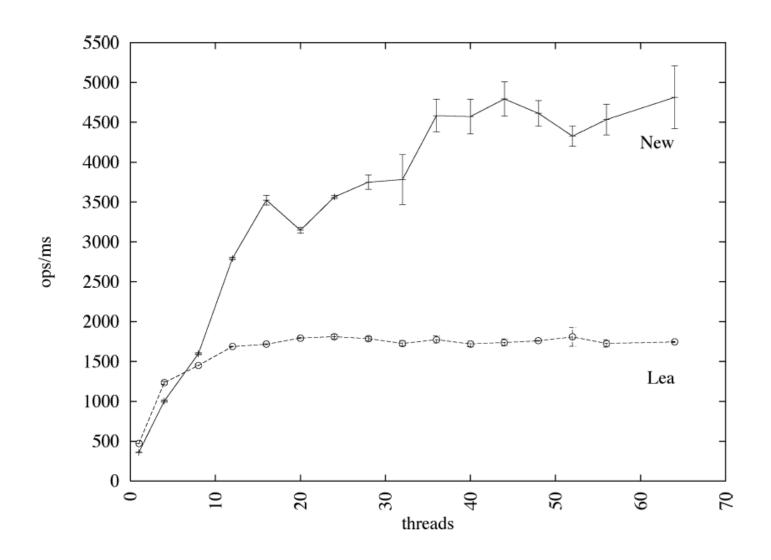
Additional Performance

- Additionally, the following parameters have been analyzed:
 - The effects of the choice of locking granularity
 - The effects of the bucket size

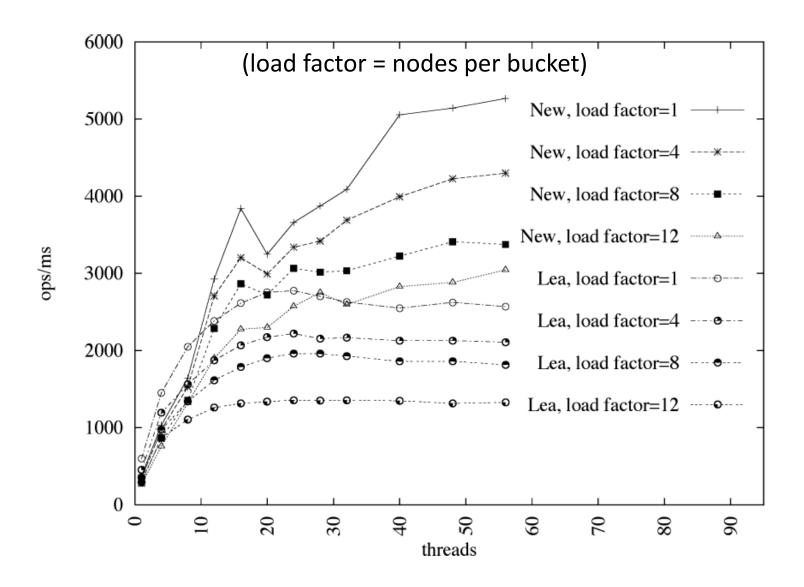
Number of Fine-Grain Locks



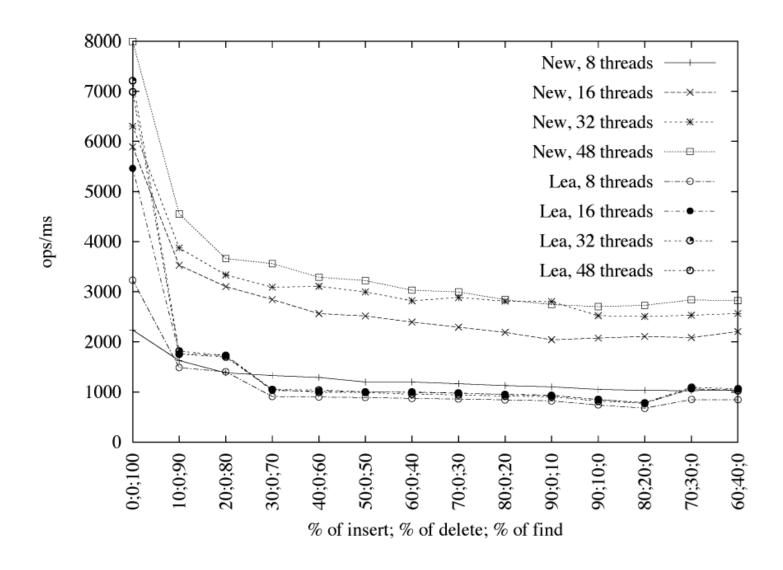
Lock-free vs. Locks



Hash Table Load Factor



Varying Operations



Summary

- We talked about techniques to deal with concurrency in linked lists
 - Hand-over-hand locking
 - Optimistic synchronization
 - Lazy synchronization
 - Lock-free synchronization
- Then we talked about hashing
 - Fine-grained locking
 - Recursive split ordering

Credits

- The first lock-free list algorithms are credited to John Valois, 1995.
- The lock-free list algorithm discussed in this lecture is a variation of algorithms proposed by Harris, 2001, and Michael, 2002.
- The lock-free hash set based on split-ordering is by Shalev and Shavit,
 2006.

That's all!

Questions & Comments?



Roger Wattenhofer