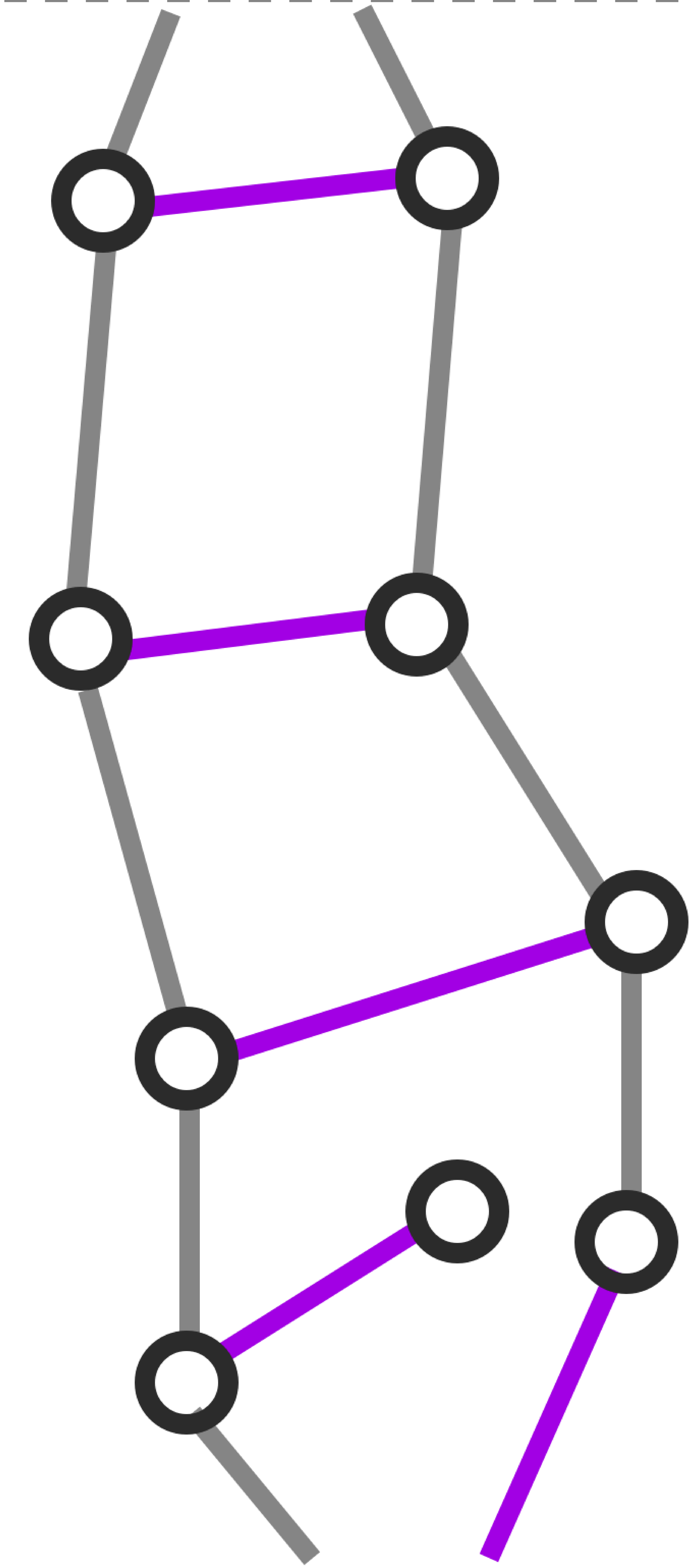
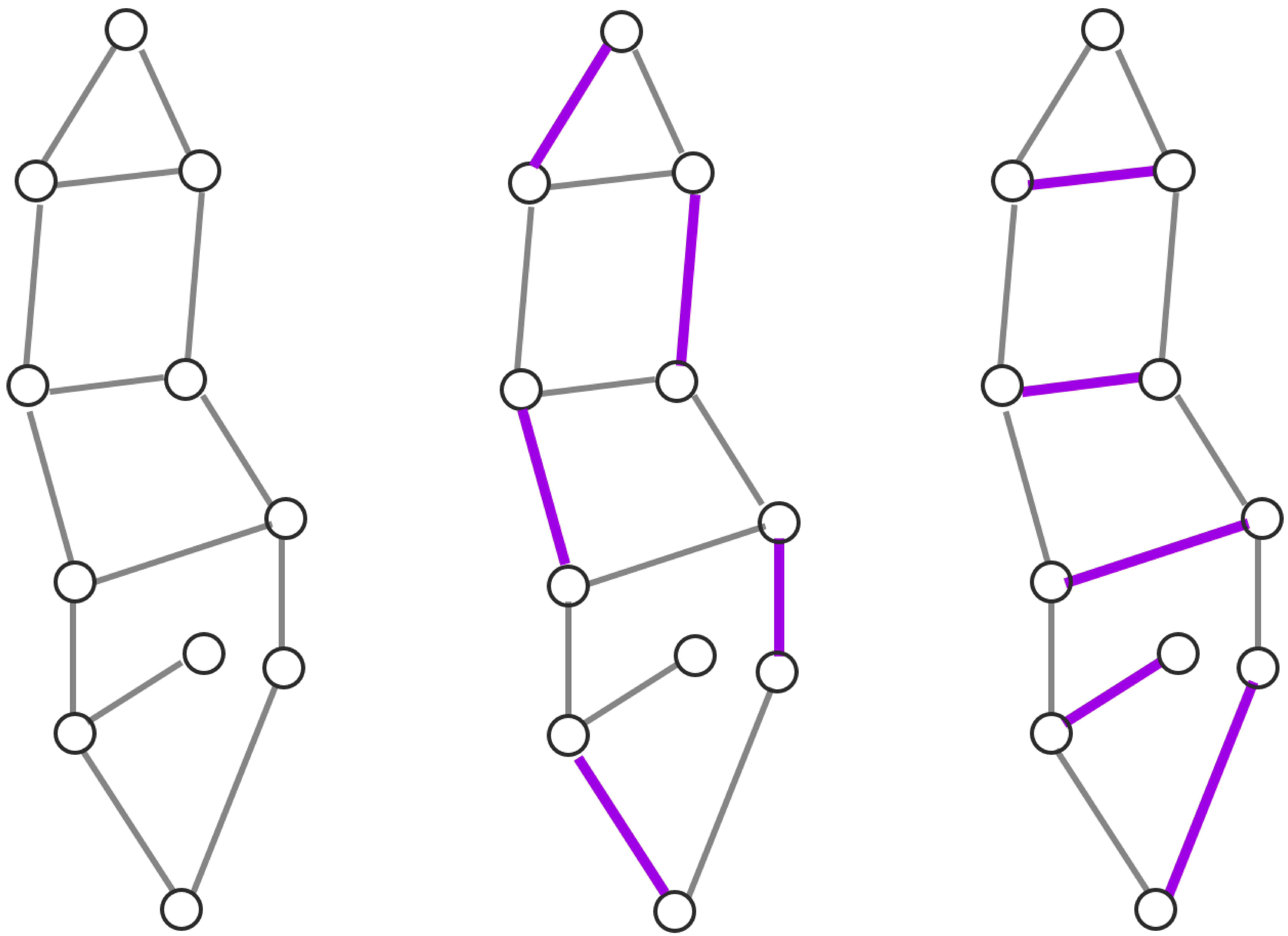


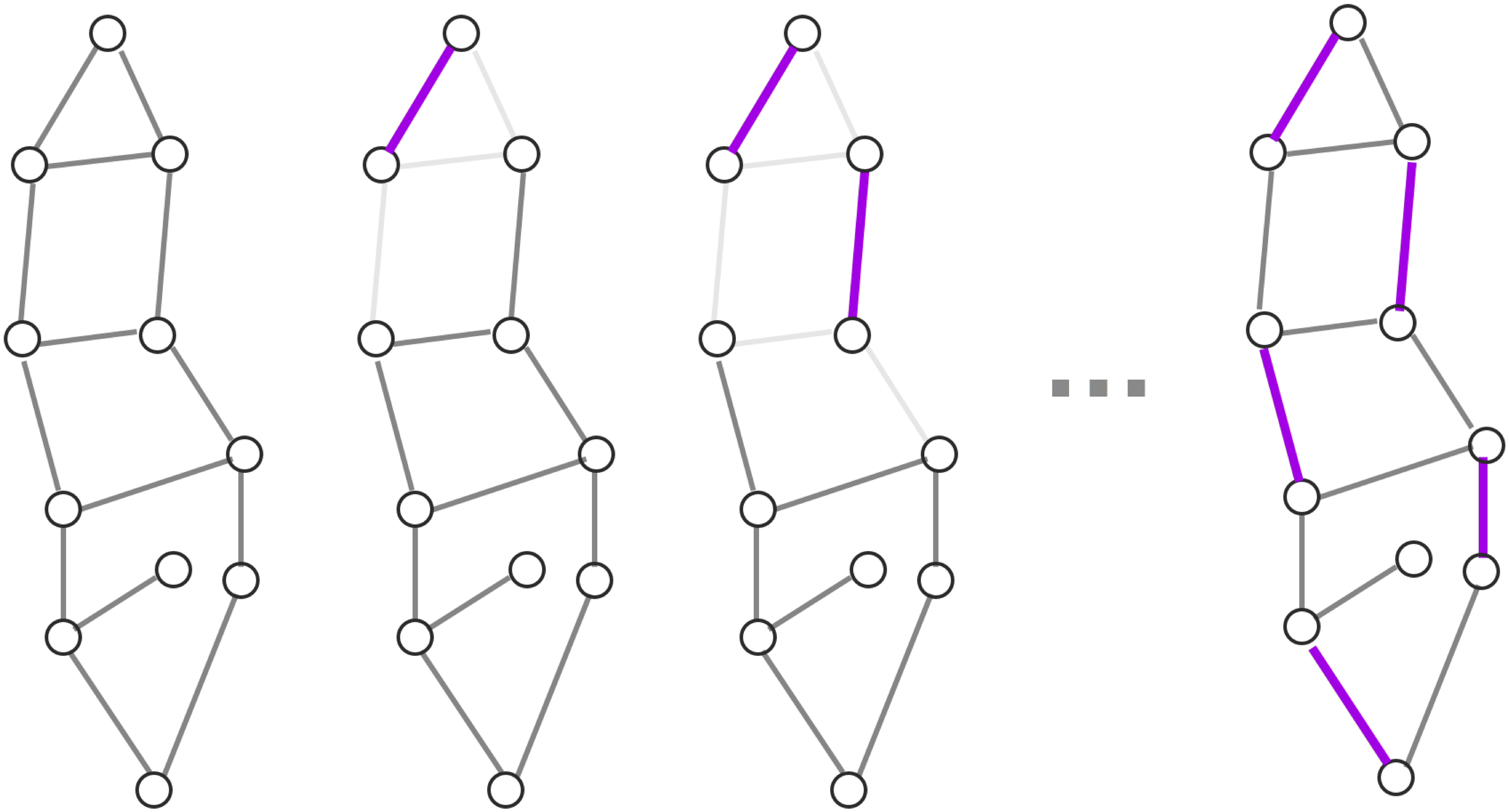
Andrea Lattuada

Maximal Matching

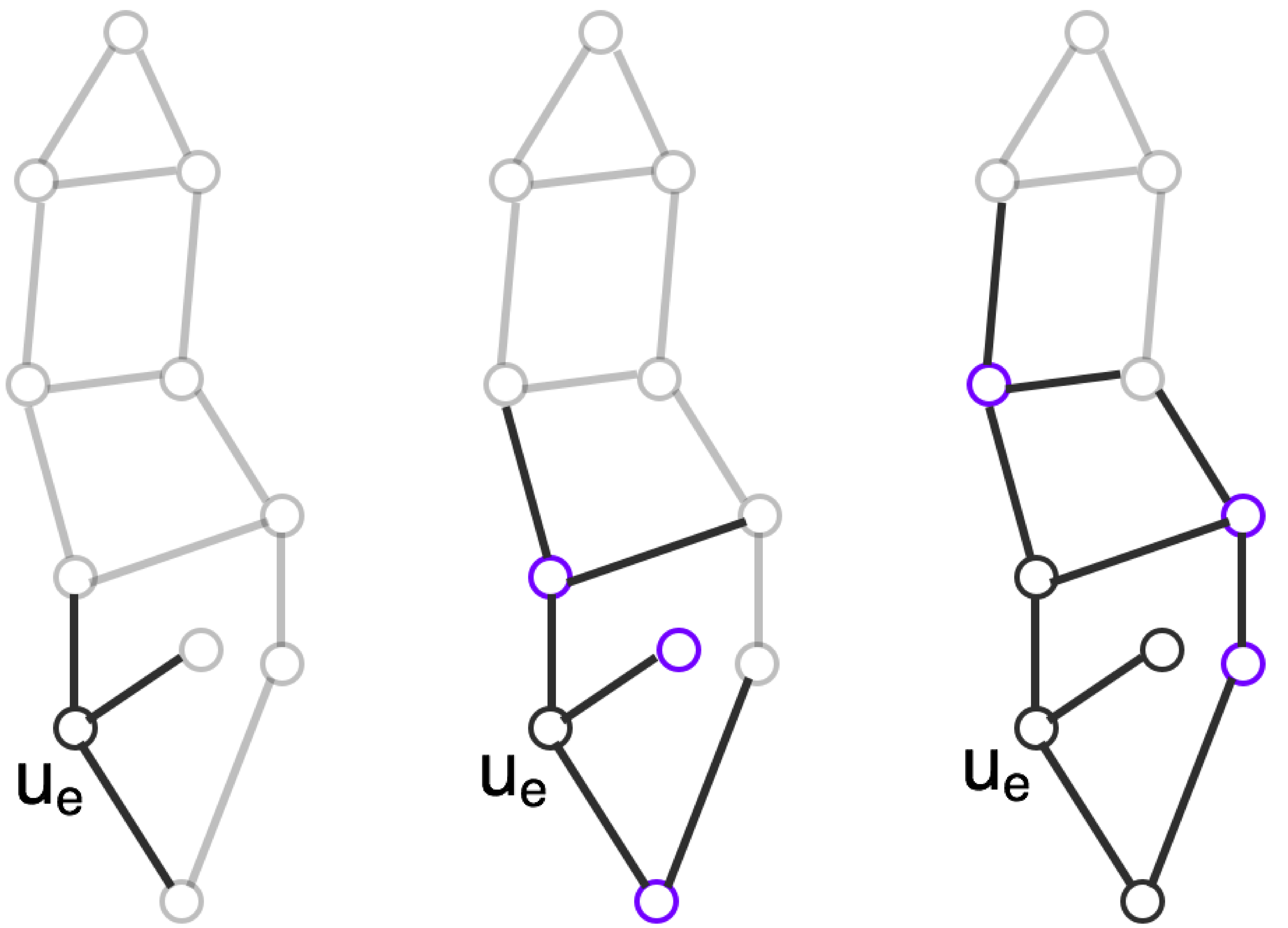


- Maximal set of vertex-disjoint edges

Being greedy



Distributed algorithm



Initially, each node only knows its incident edges

Nodes exchange messages to learn more about other nodes and edges

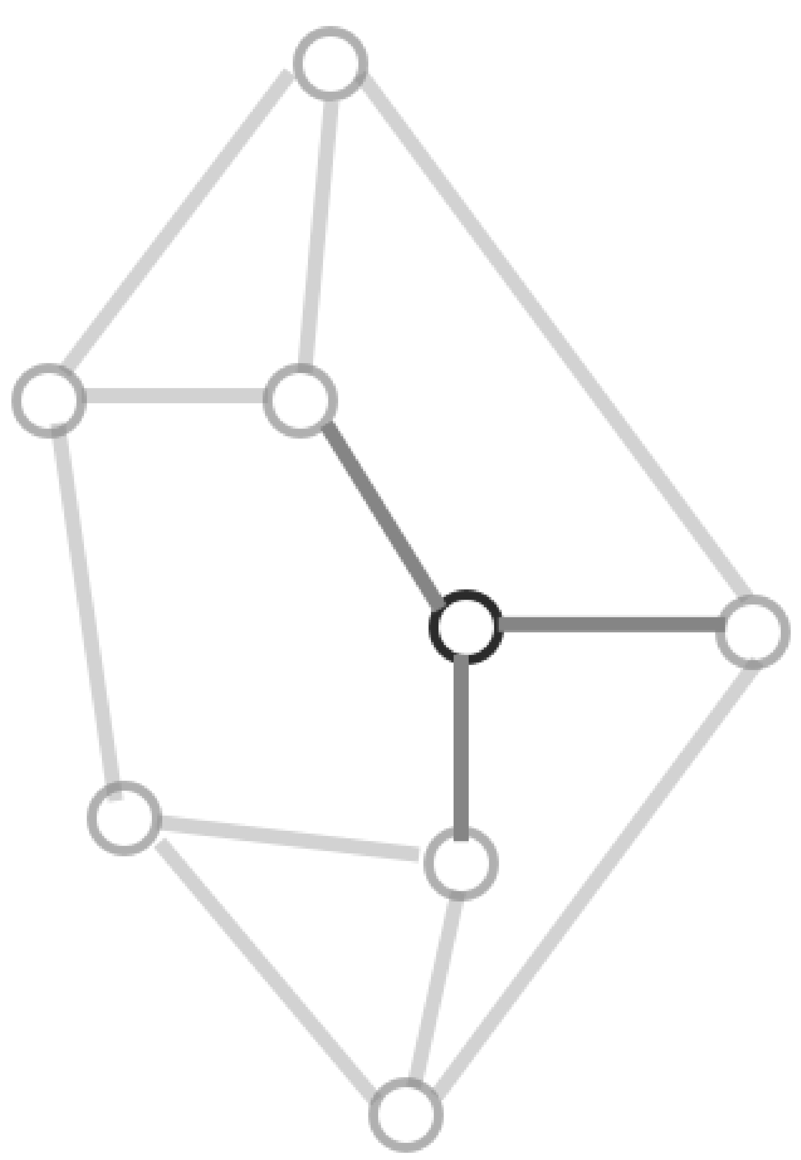
- Time = number of communication rounds

Known bounds

nodes with ids

$$O(\Delta + \log^* n)$$

$$\Omega(\mathrm{polylog}(\Delta) + \log^* n)$$



Δ Degree

- Max number of edges incident to a node

Known bounds

non-local

$$O(\Delta + \log^* n)$$

$$\Omega(\text{polylog}(\Delta) + \log^* n)$$

$$\log^* n := \begin{cases} 0 & \text{if } n \leq 1 \\ 1 + \log^*(\log n) & \text{if } n > 1 \end{cases}$$

Closing the gap

$$O(\Delta + \log^* n)$$

exponential!

$$\Omega(\text{polylog}(\Delta) + \log^* n)$$

Closing the gap

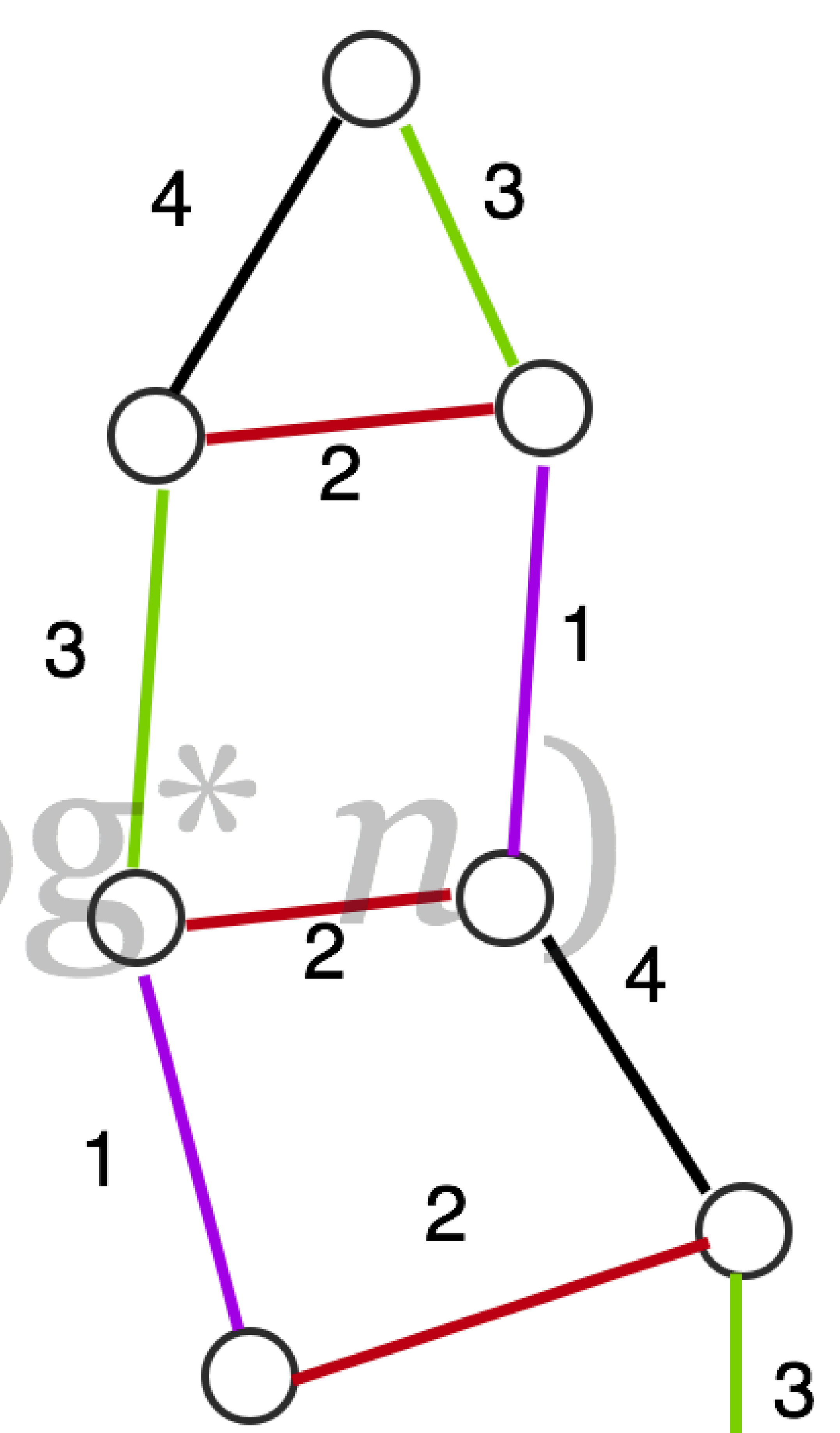
$O(\Delta + \log^* n)$

(╯°□°）╯︵ ┻━┻ in general graphs

exponential!

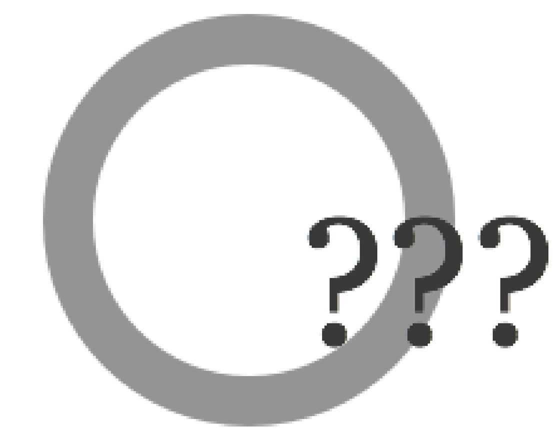
simpler model

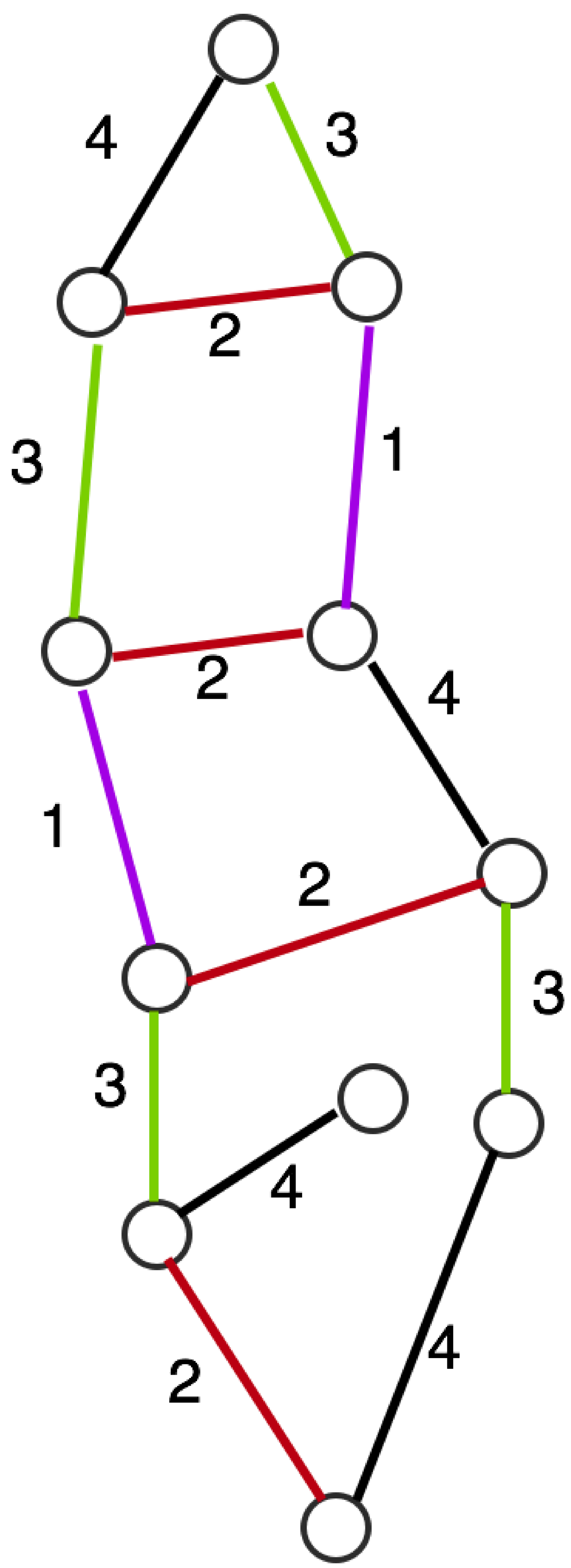
$\Omega(\text{polylog}(\Delta) + \log^* n)$



anonymous, k-edge-colored

nodes without ids

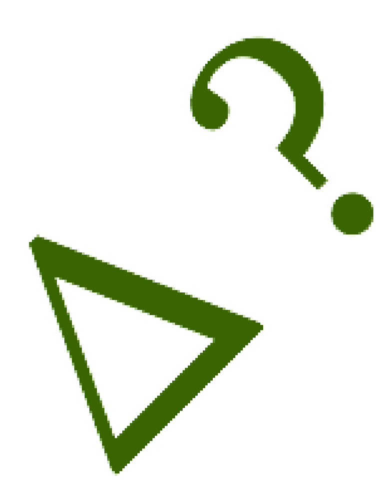




- no two edges incident to the same node share the same color
- **at most**, k colors

anonymous, k-edge-colored

$$O(\Delta + \log^* k)$$



$$\Omega(\log^* k)$$

anonymous,
k-edge-colored

$O(\Delta + \log^* k)$

thight bound for distributed maximal matching
in anonymous, k-edge-colored graphs

this work

$\Omega(\Delta)$

previous work

$\Omega(\log^* k)$

$\therefore \quad \Omega(\Delta + \log^* k)$


Δ ≤ k
k colors
degree Δ
this work
Ω(Δ)
1
4
2
3
Δ ≤ k
Δ
k
Ω(k) ⇒ Ω(Δ)

Theorem 1

this work

$\Omega(k)$

Let k be a positive integer. A deterministic distributed algorithm that finds a maximal matching in any anonymous, k-edge-colored graph requires at least k - 1 communication rounds

$\Delta \leq k$

$\Omega(k) \Rightarrow \Omega(\Delta)$

$$\Omega(k - 1) \Rightarrow \Omega(\Delta) \Rightarrow \Omega(\Delta + \log^* k)$$

$\Omega(\log^* k)$

Theorem 1

this work

$\Omega(k-1)$

Let k be a positive integer. A deterministic distributed algorithm that finds a maximal matching in any anonymous, k-edge-colored graph requires at least k - 1 communication rounds

Greedy

$\Delta <= k$

Deterministic distributed greedy algorithm
to find a maximal matching
on a k-edge-colored anonymous graph

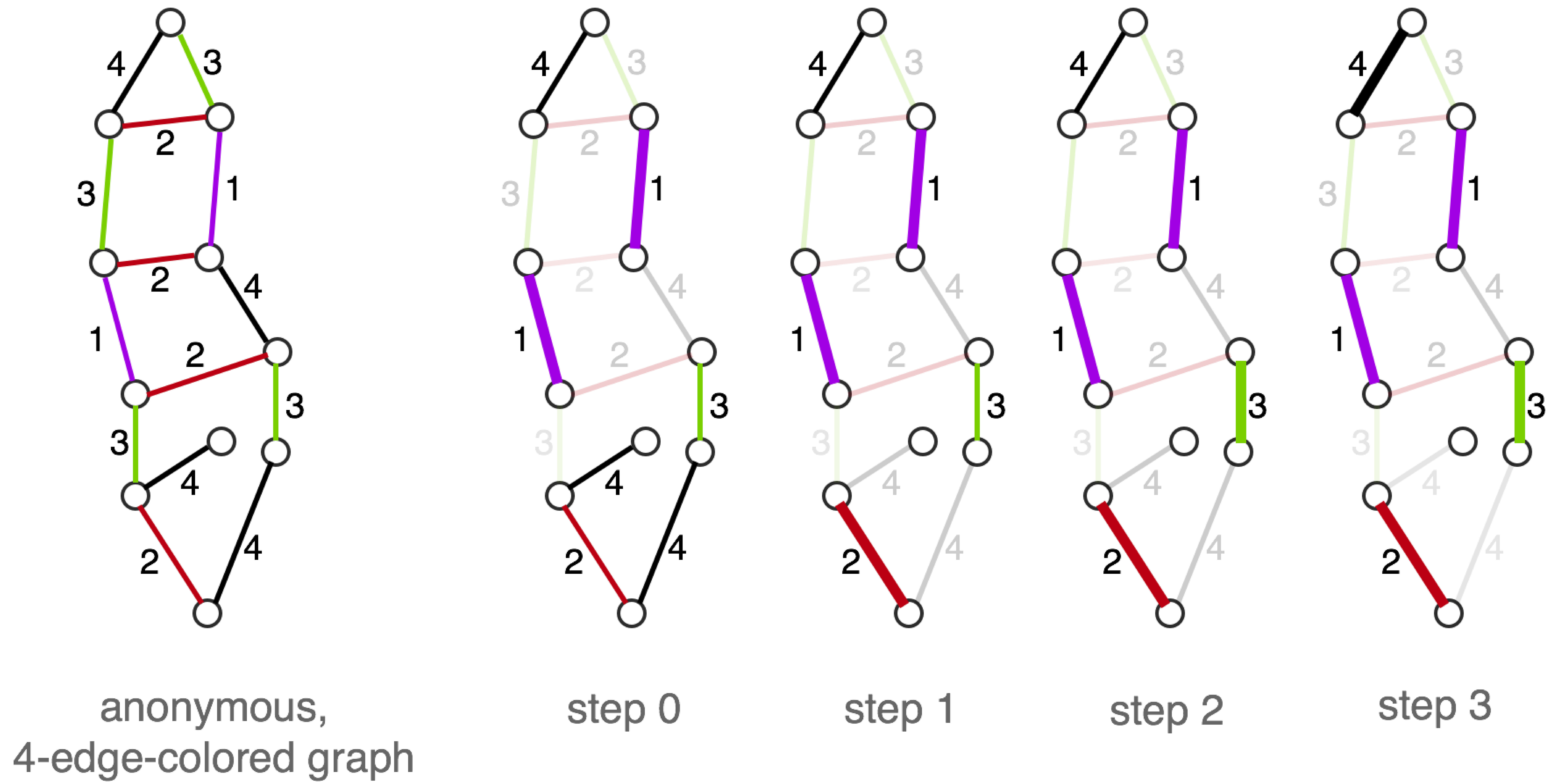
$\Omega(k) => \Omega(\Delta)$

$\Omega(k-1) => \Omega(\Delta) => \Omega(\Delta + \log^* k)$

$\Omega(\log^* k)$

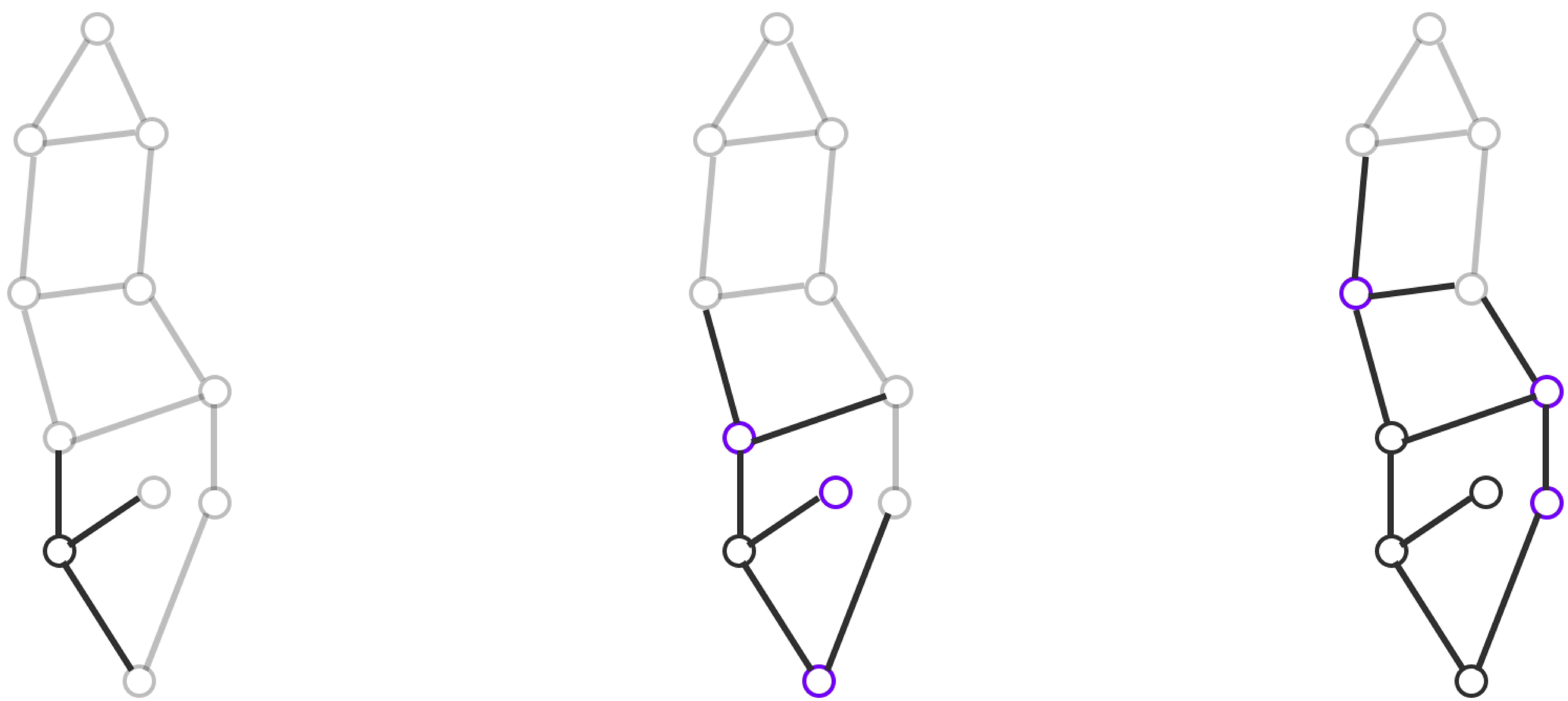
Deterministic distributed greedy algorithm
to find a maximal matching
on a k-edge-colored anonymous graph

this work
$\Omega(k-1)$



radius-k neighbourhoods

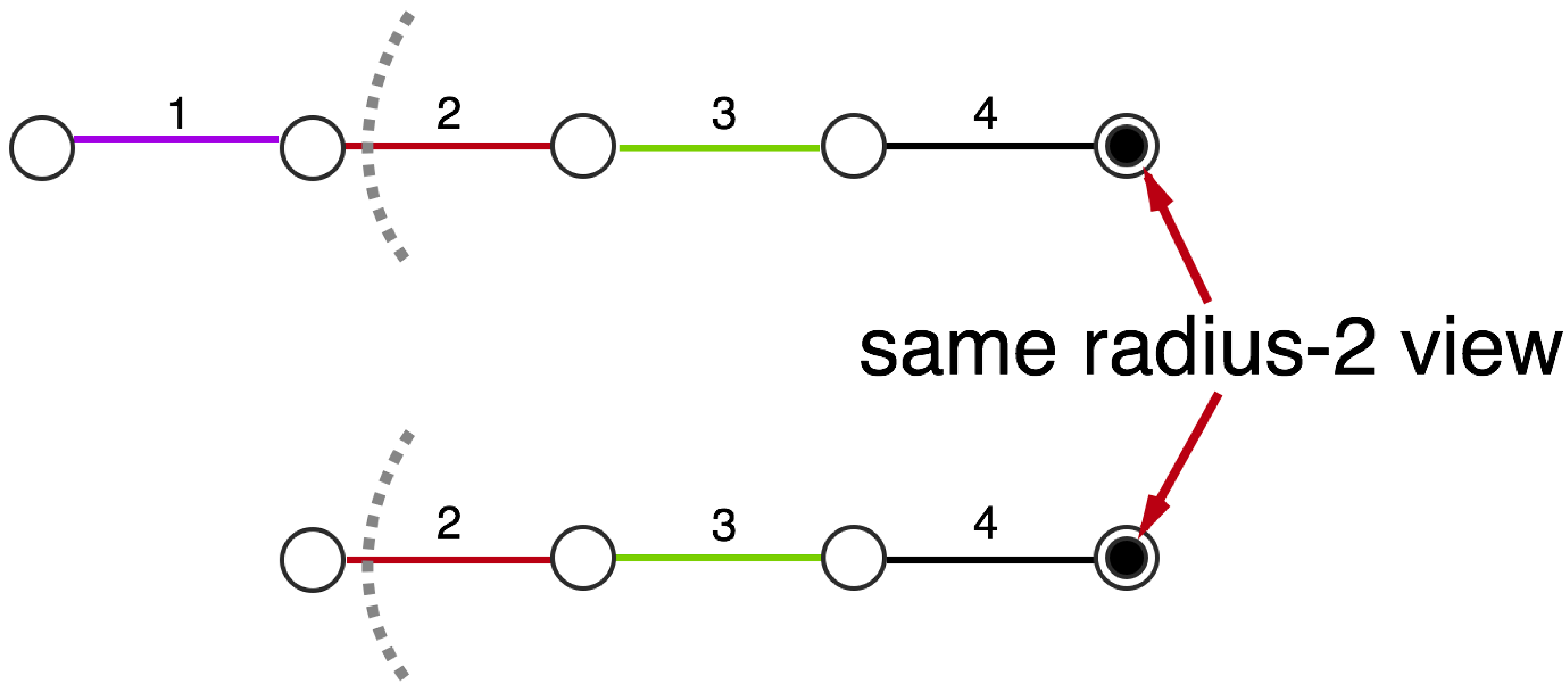
Initially, each node only knows its incident edges, **its radius-0 neighbourhood**



radius-0 neighbourhood **radius-1 neighbourhood** **radius-2 neighbourhood**

Deterministic distributed greedy algorithm
to find a maximal matching
on a k-edge-colored anonymous graph

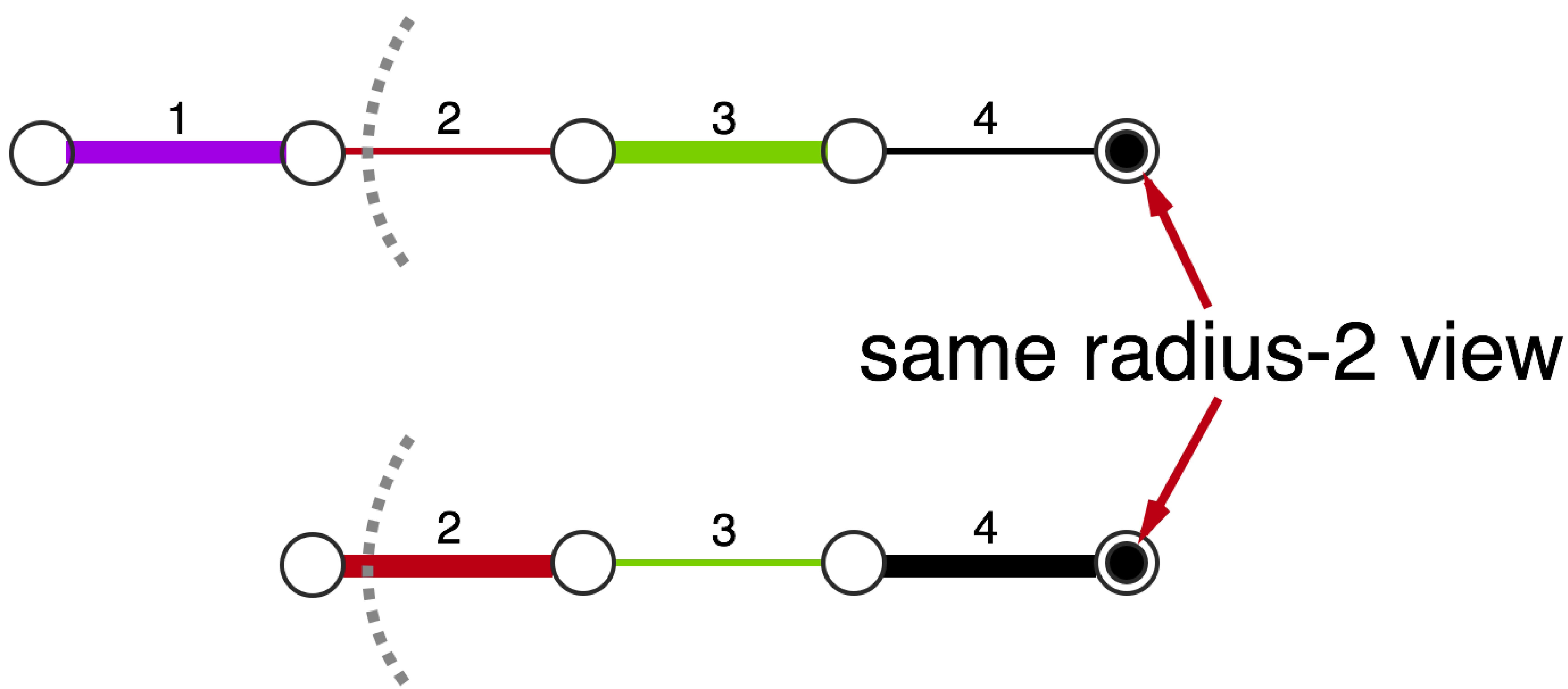
this work
$\Omega(k-1)$



after 2 communication rounds

Deterministic distributed greedy algorithm
to find a maximal matching
on a k-edge-colored anonymous graph

this work
$\Omega(k-1)$



to get different (local) output

need one more
communication round

this greedy algorithm
$\Rightarrow \Theta(k-1)$

greedy algorithm

$\Theta(k-1)$

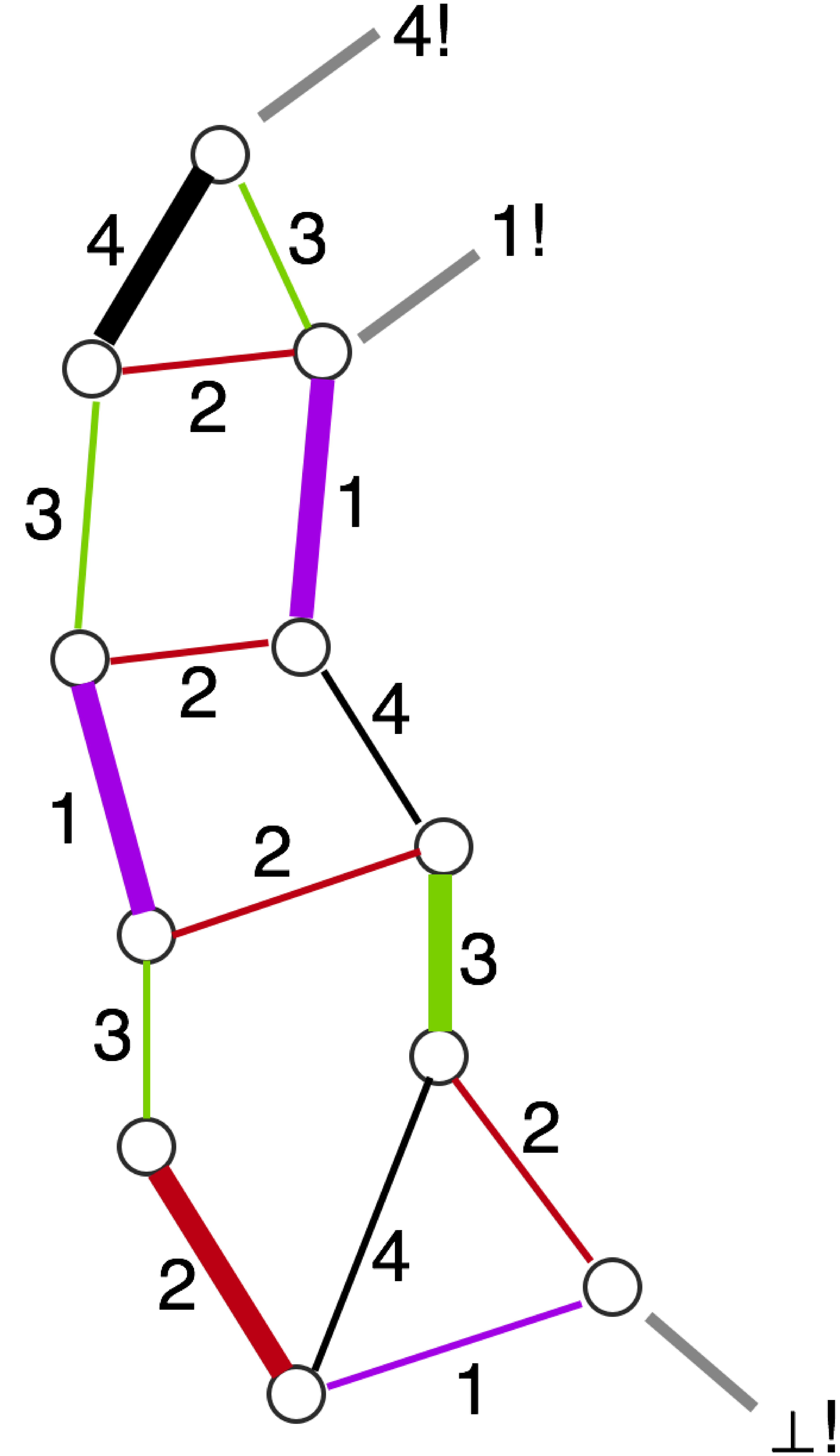
Tight lower bound for deterministic distributed maximal matching on a k-edge-colored graph

back to

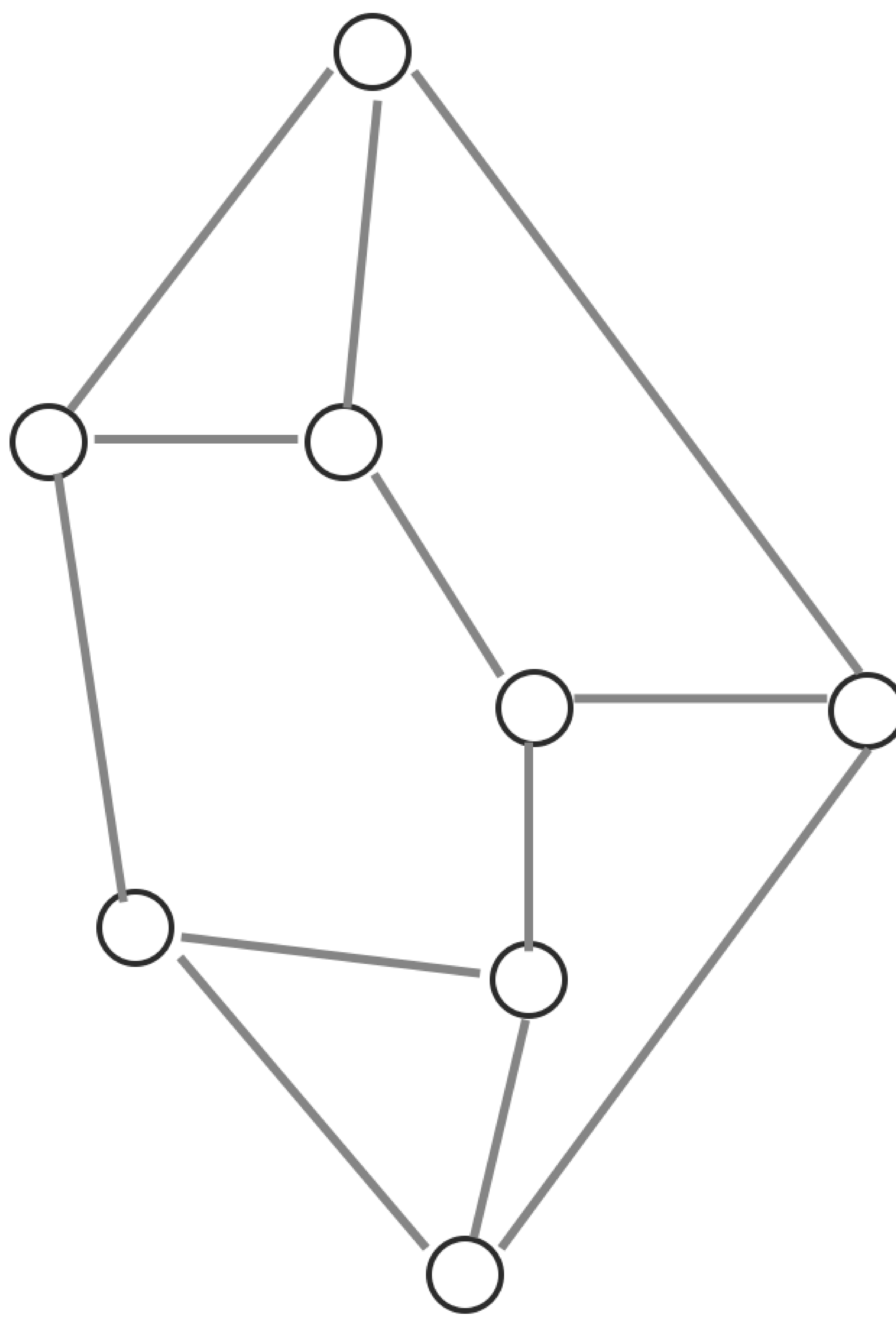
this work

$\Omega(k-1)$

Local output



d-regular graph



3-regular

greedy algorithm

$\Theta(k - 1)$

this work

$\Omega(k - 1)$

Theorem 1

Let k be a positive integer. A deterministic distributed algorithm that finds a maximal matching in any anonymous, k-edge-colored graph requires at least k - 1 communication rounds

greedy algorithm

$\Theta(k - 1)$

this work

$\Omega(k - 1)$

Theorem 1

Let k be a positive integer. A deterministic distributed algorithm that finds a maximal matching in any anonymous, k-edge-colored graph requires at least k - 1 communication rounds

Theorem 2

Let **k >= 3** and **d = k - 1**

Assume a distributed algorithm that finds a maximal matching in any d-regular k-color graph.
Then there are two d-regular k-colored graphs A, B such that a node u_e has the same (d - 1)-radius view in A and B and u_e is unmatched in A and matched in B

this work

$\Omega(k-1)$

Building a worst case

two d-regular k-colored graphs A, B such that a node u_e has the same d-radius view in A and B and u_e is unmatched in A and matched in B

k-colors, d-regular

$d = k - 1$

k >= 3 and **d = k - 1**

this work

$\Omega(k - 1)$

Group

Generators = { 1, 2, ..., k }

Operation: concatenation

1 . 3 = 13

32 . 1 = 321

Identity element: *e*

Inverse

1 . 1 = *e*

21 . 1 = 2 . e = 2

342 . 213 = 3413

Associativity

k >= 3 and d = k - 1

this work

$\Omega(k-1)$

Group

Generators = { 1, 2, ..., k }

Operation: concatenation

1 . 3 = 13

32 . 1 = 321

Identity element: *e*

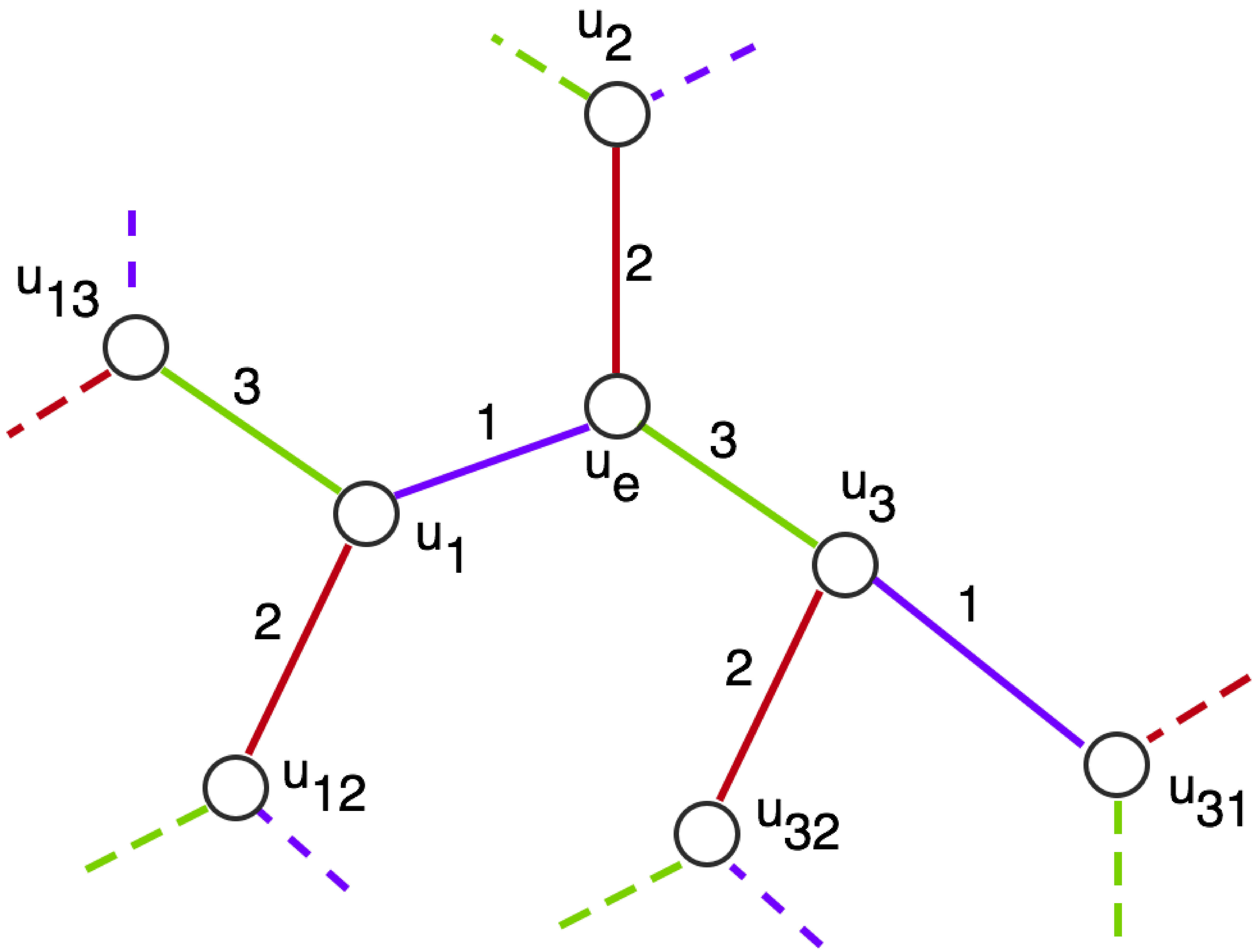
Inverse

1 . 1 = *e*

21 . 1 = 2 . e = 2

342 . 213 = 3413

Associativity



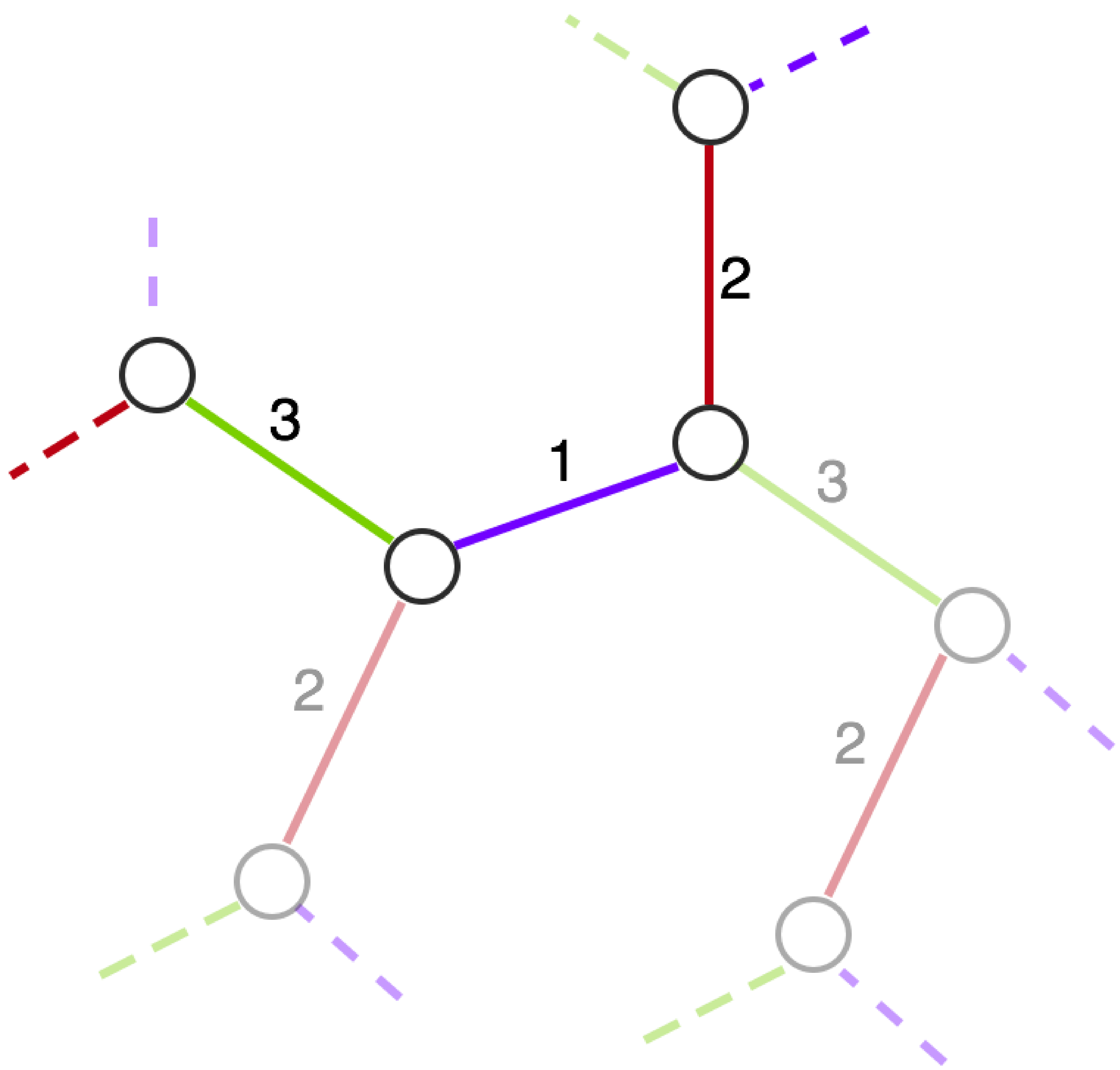
k >= 3 and d = k - 1

this work

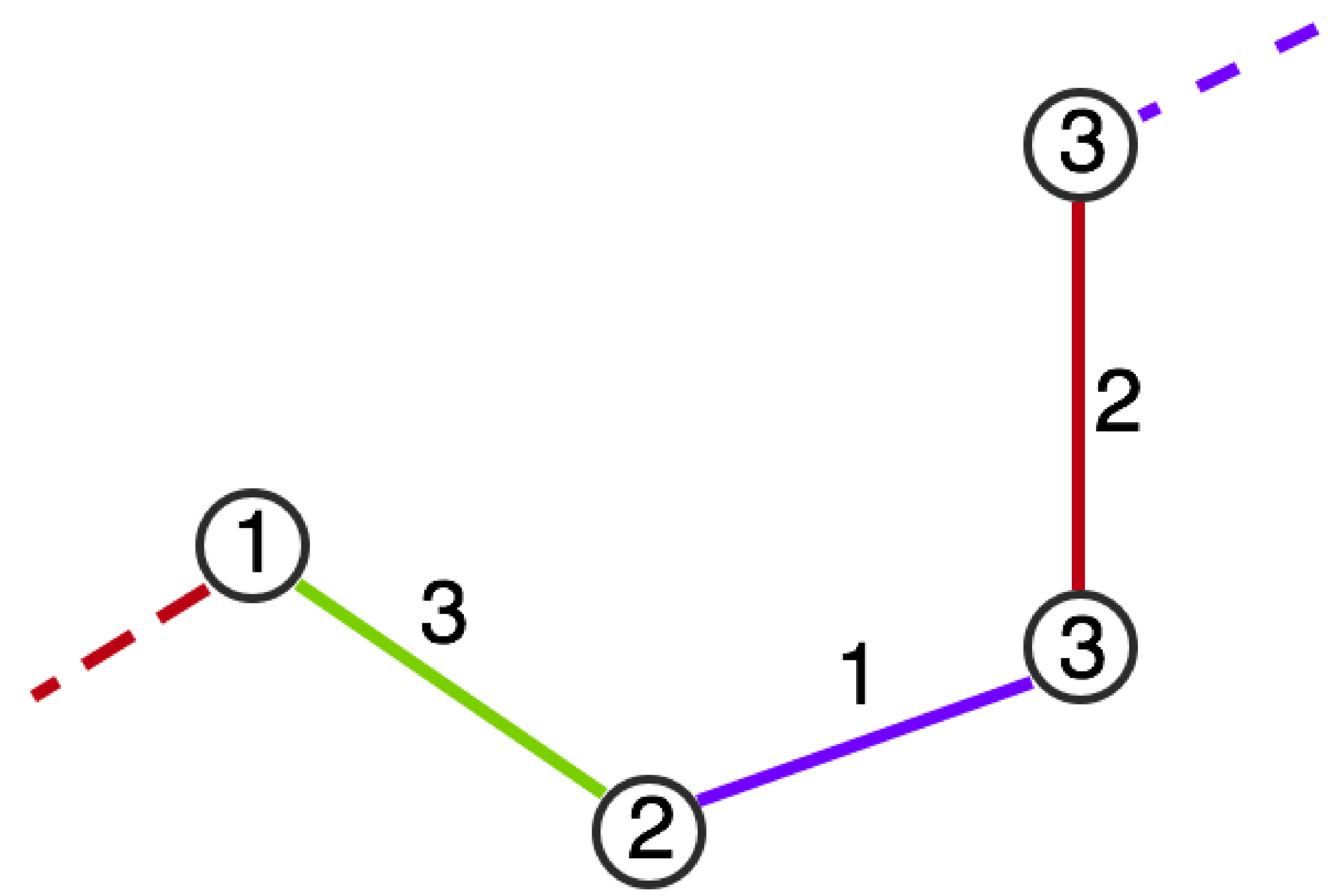
$\Omega(k - 1)$

Forbidden color

d-regular, k-color; d = k - 1



3-regular, 3-color



2-regular, 3-color

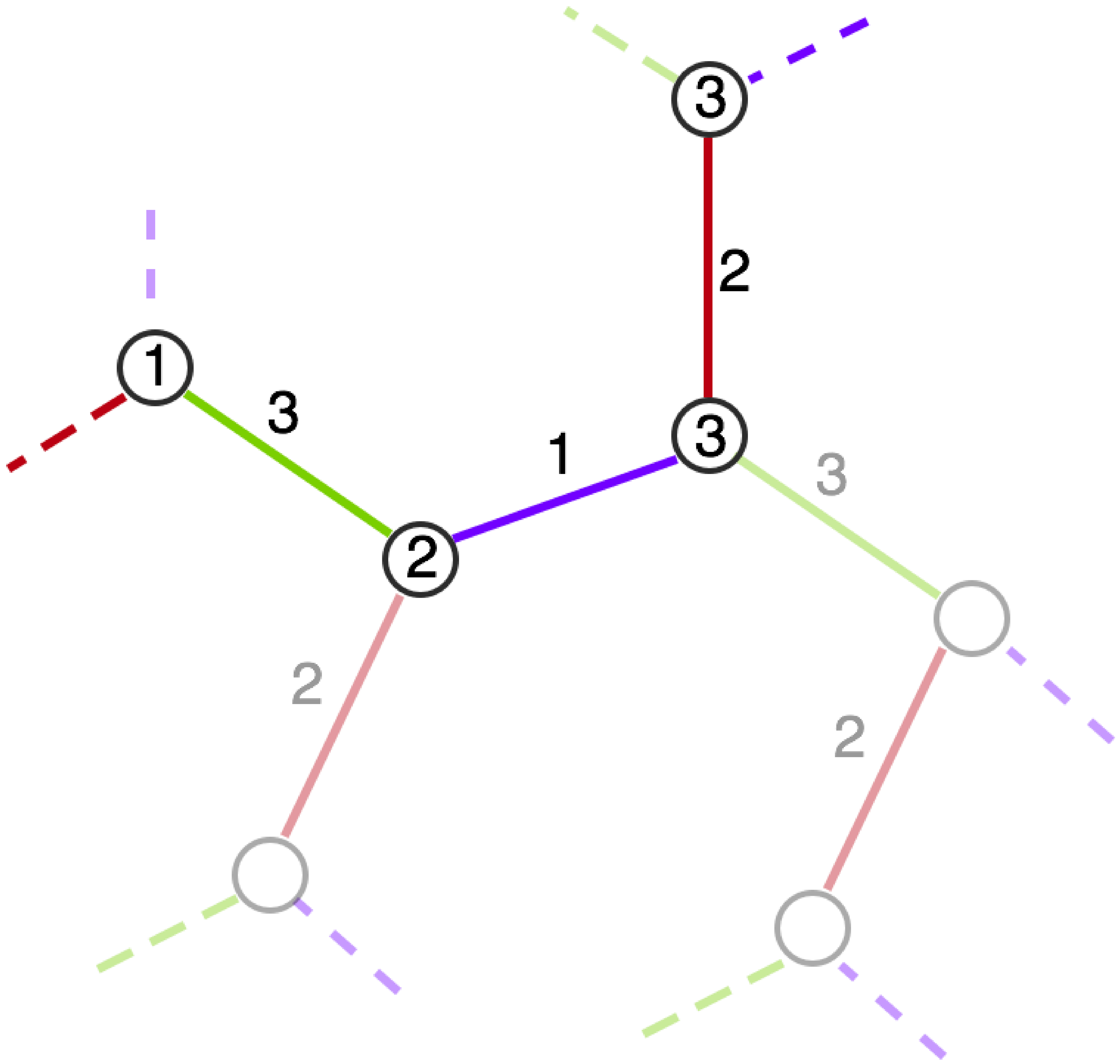
k >= 3 and d = k - 1

this work

$\Omega(k - 1)$

Worst case graphs

two d-regular k-colored graphs A, B such that a node *e* has the same d-radius view in A and B and u_e is unmatched in A and matched in B

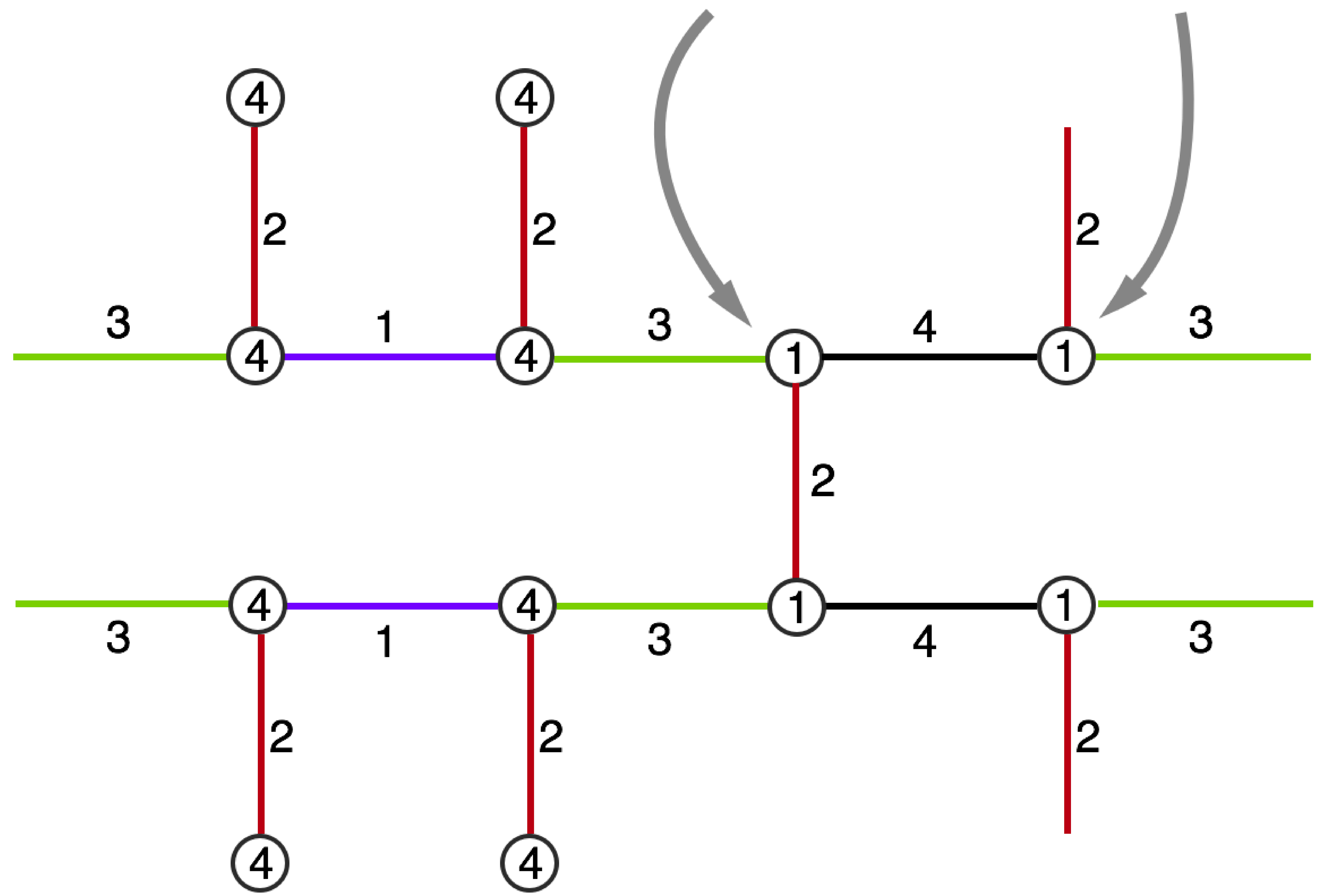


Simplifying the graph

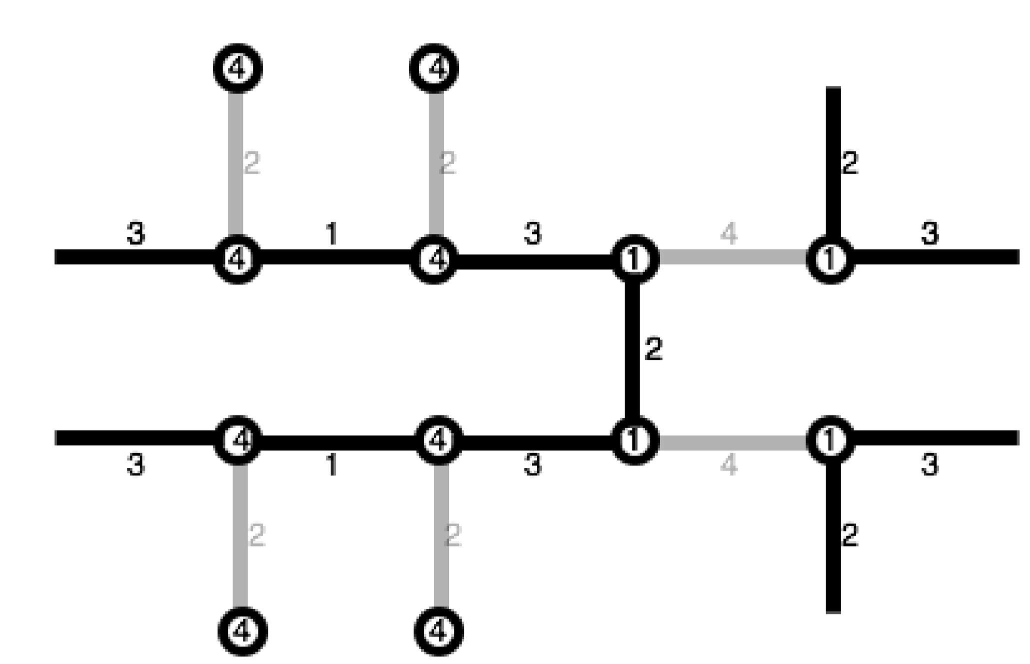
this work
$\Omega(k-1)$

leveraging simmetry

same radius-∞ view



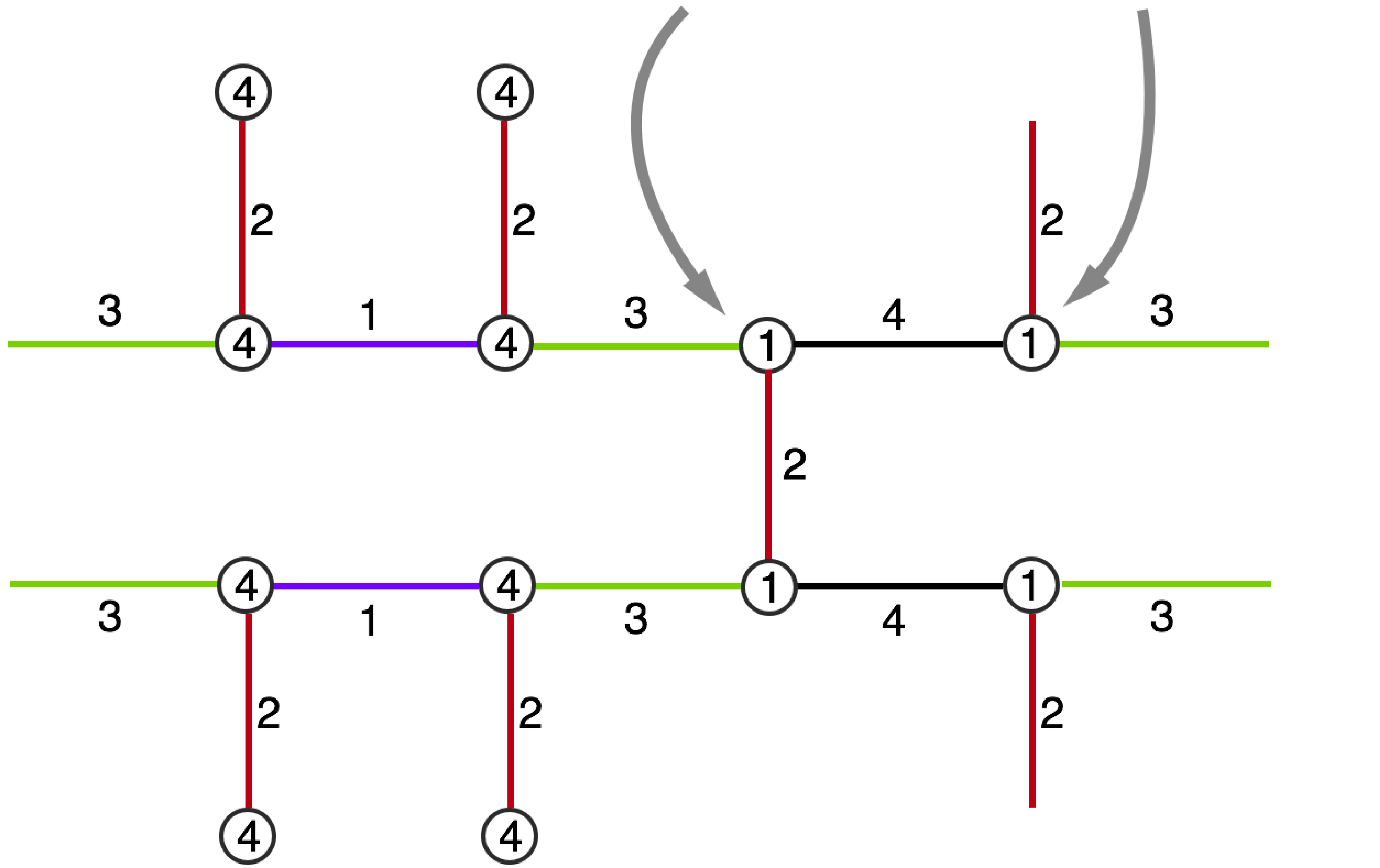
3-regular, 4-color

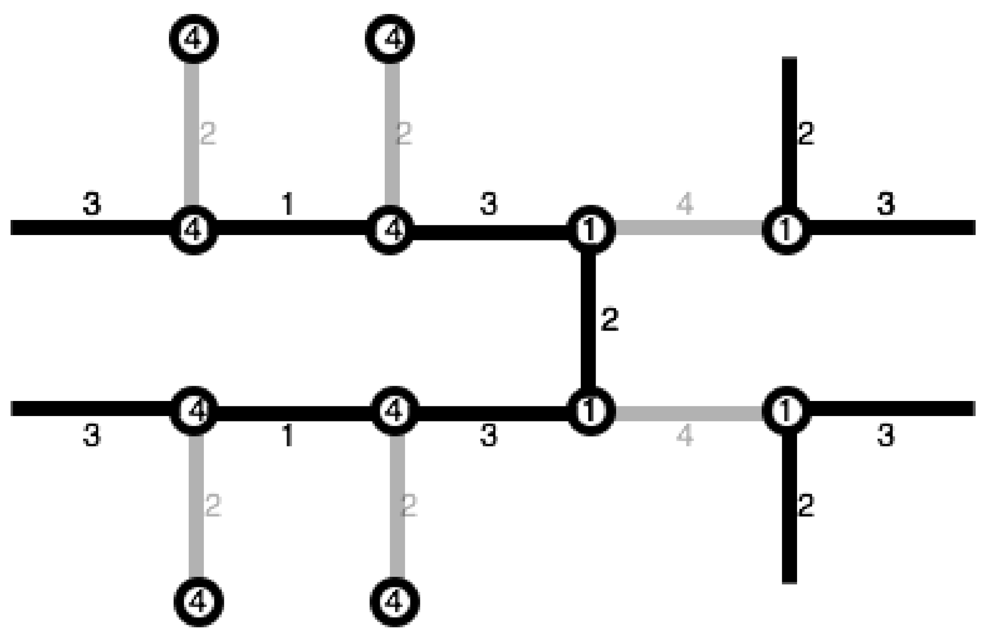


Simplifying the graph

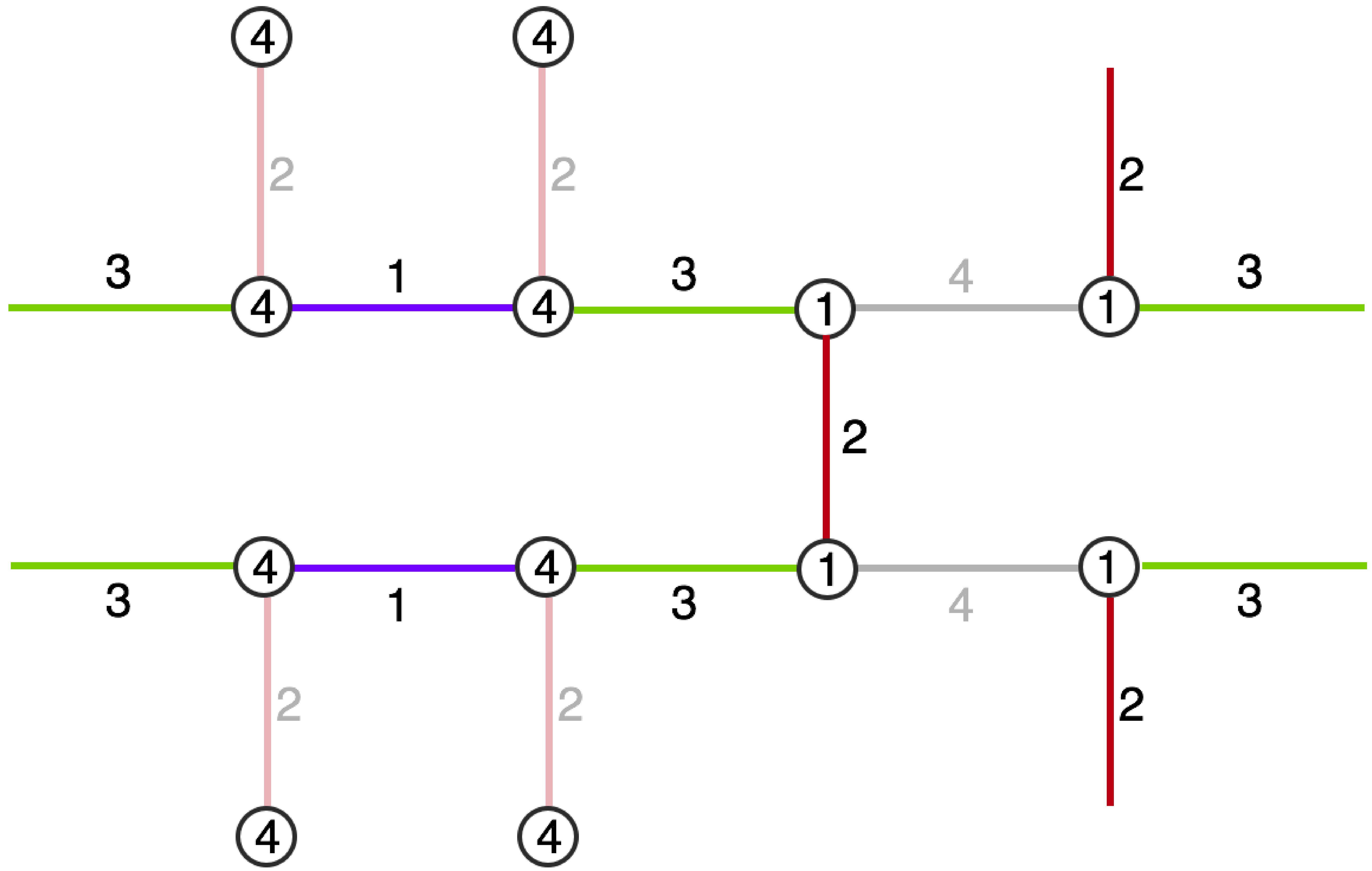
leveraging simmetry

same radius-∞ view





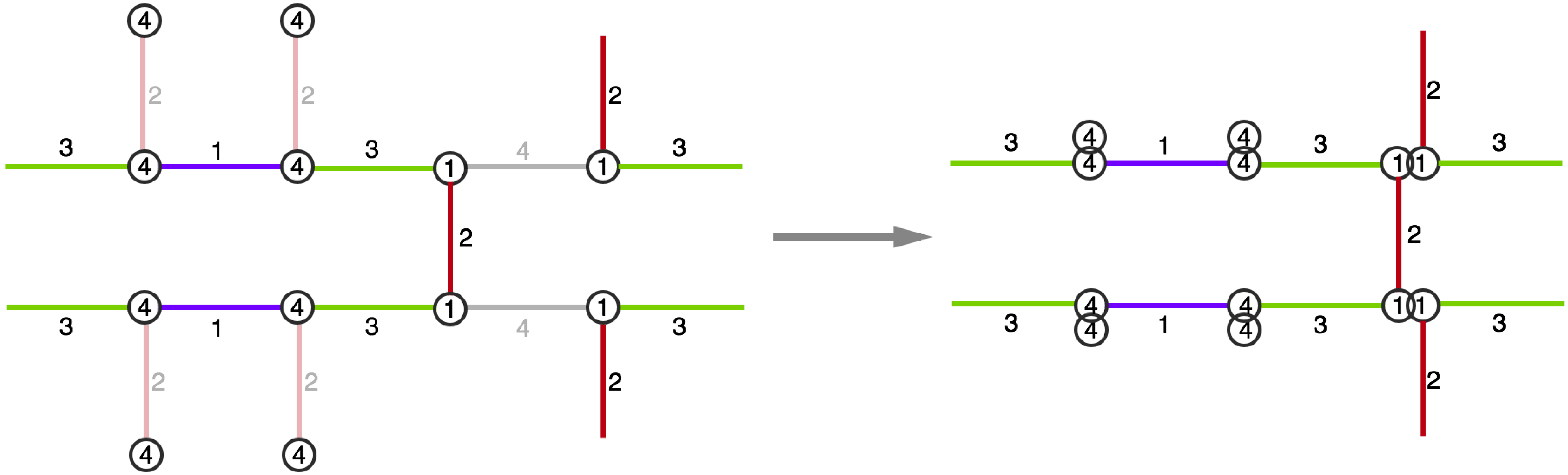
Templates



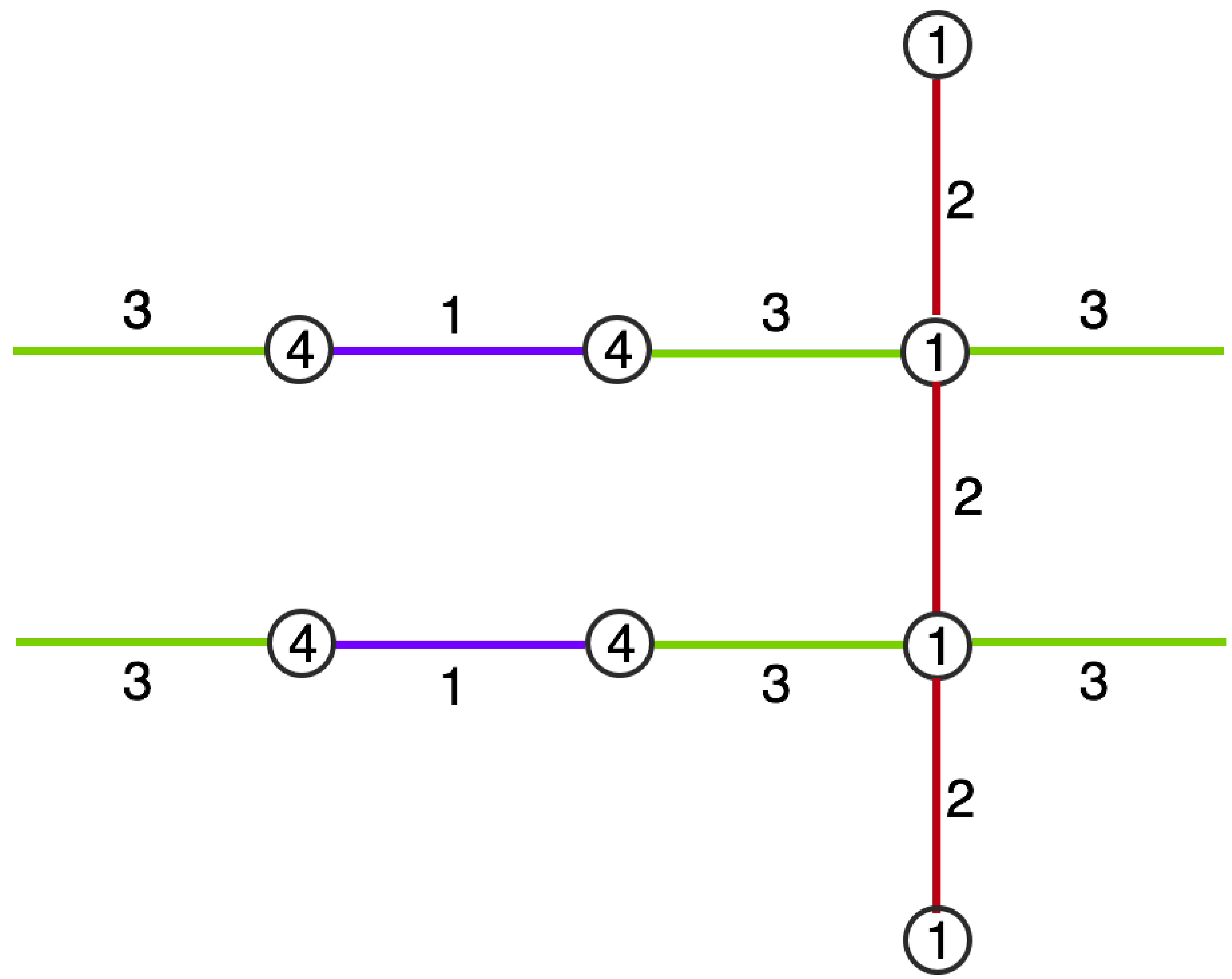
3-regular, 4-color

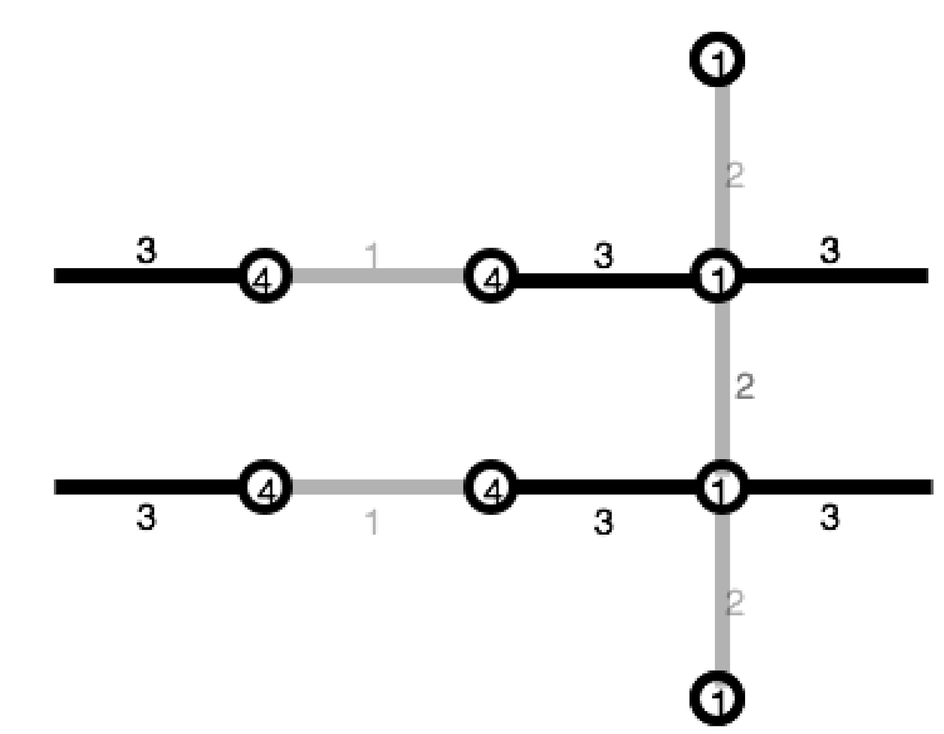
Templates


4
3
1

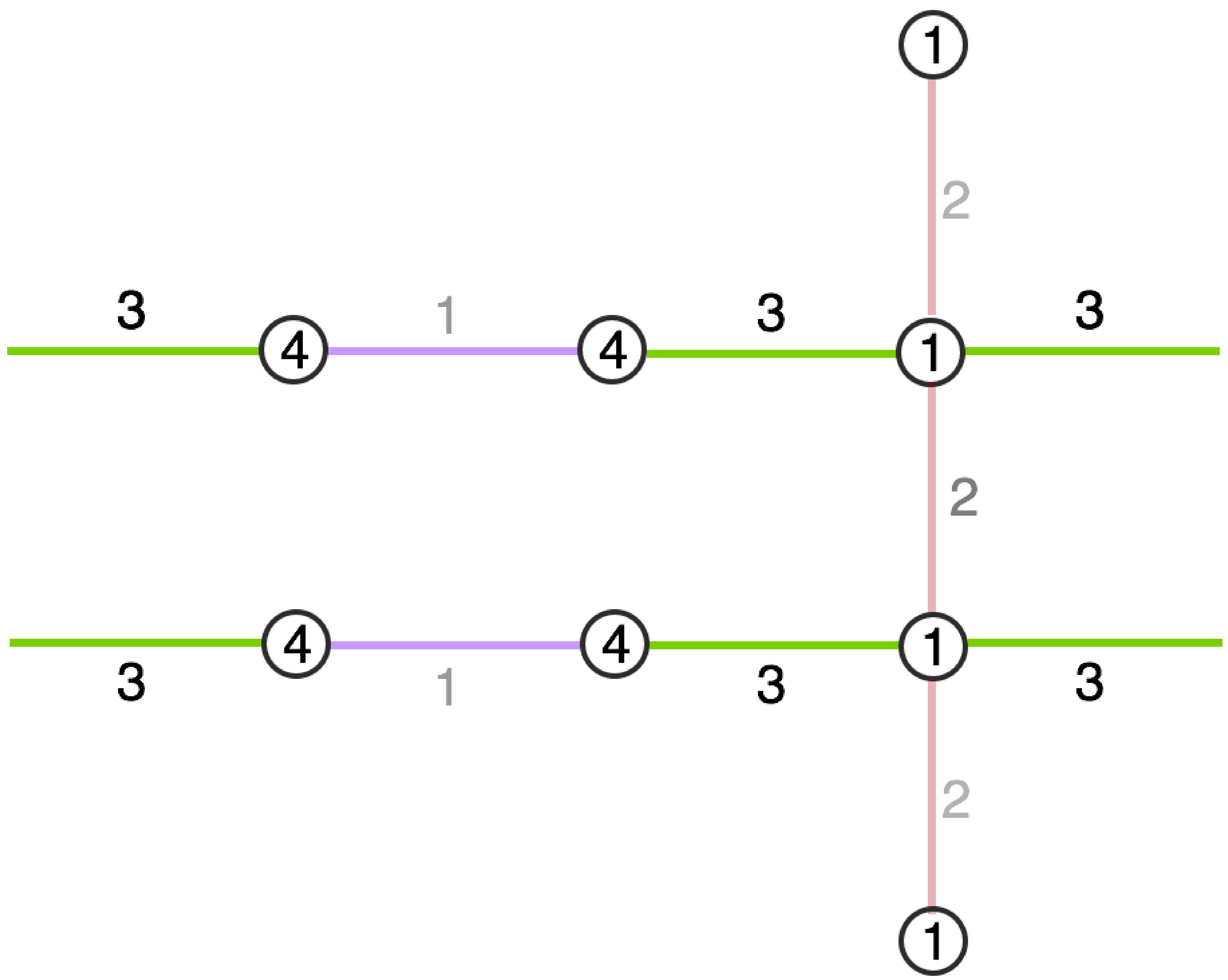

4 4 2 2 2
3 4 1 4 3 1 4 1 3
2
4 1 4 3 1 4 1
3 1 3 4 3
2 2 2
4 4
4 4 2
3 4 1 4 3 1 1 3
2
4 1 4 3 1 1
3 4 1 4 3 3
2

Templates

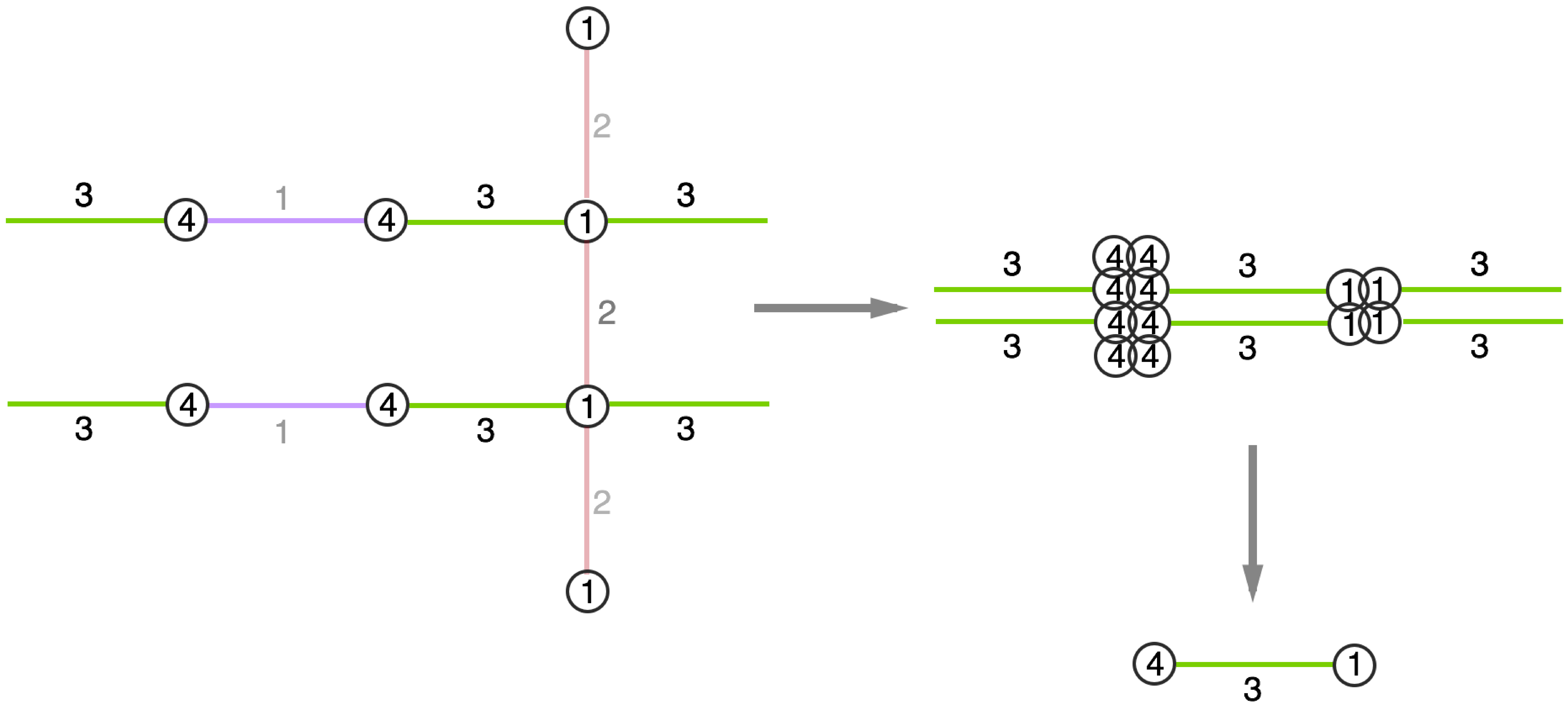




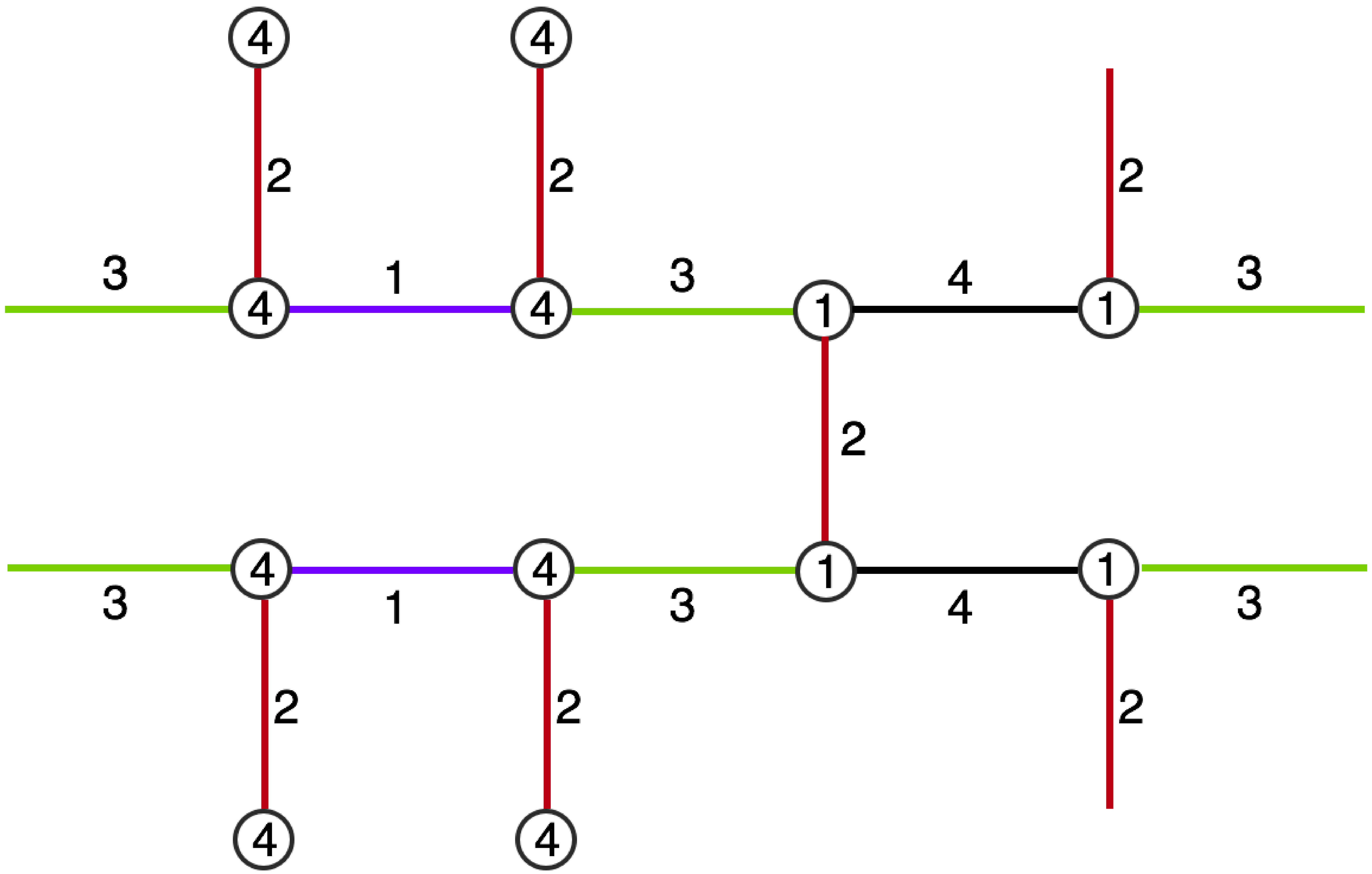
Templates



Templates



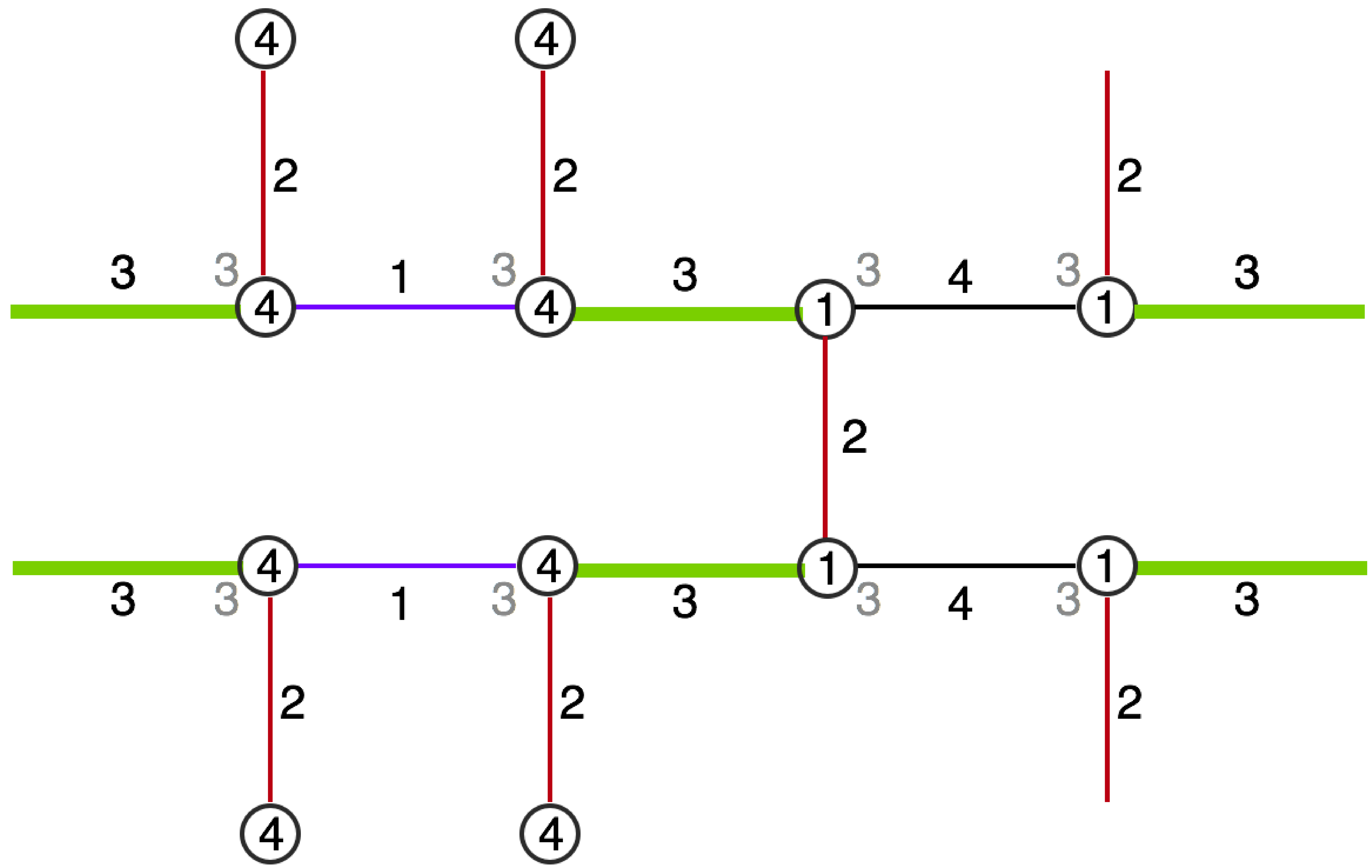
Templates

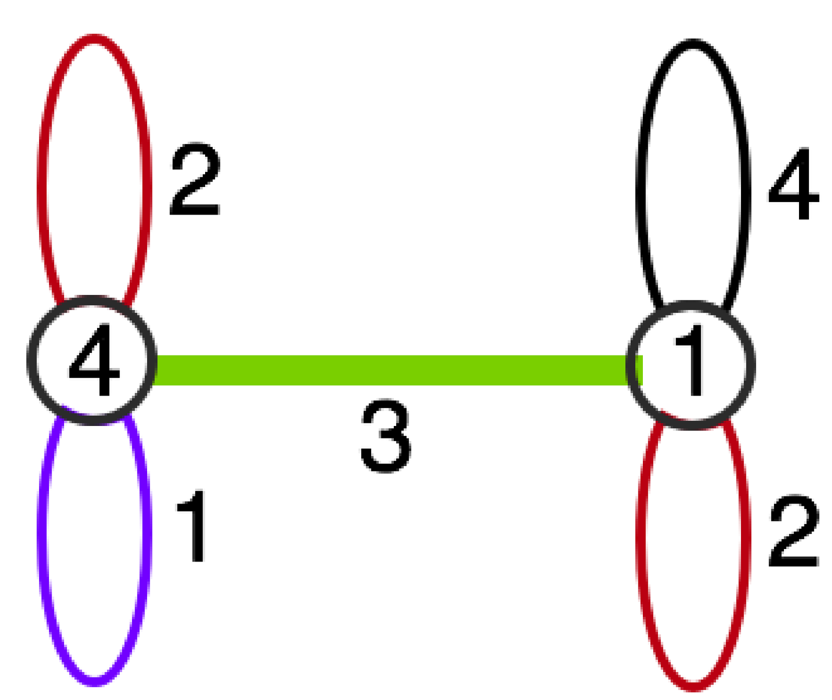


Templates

this work

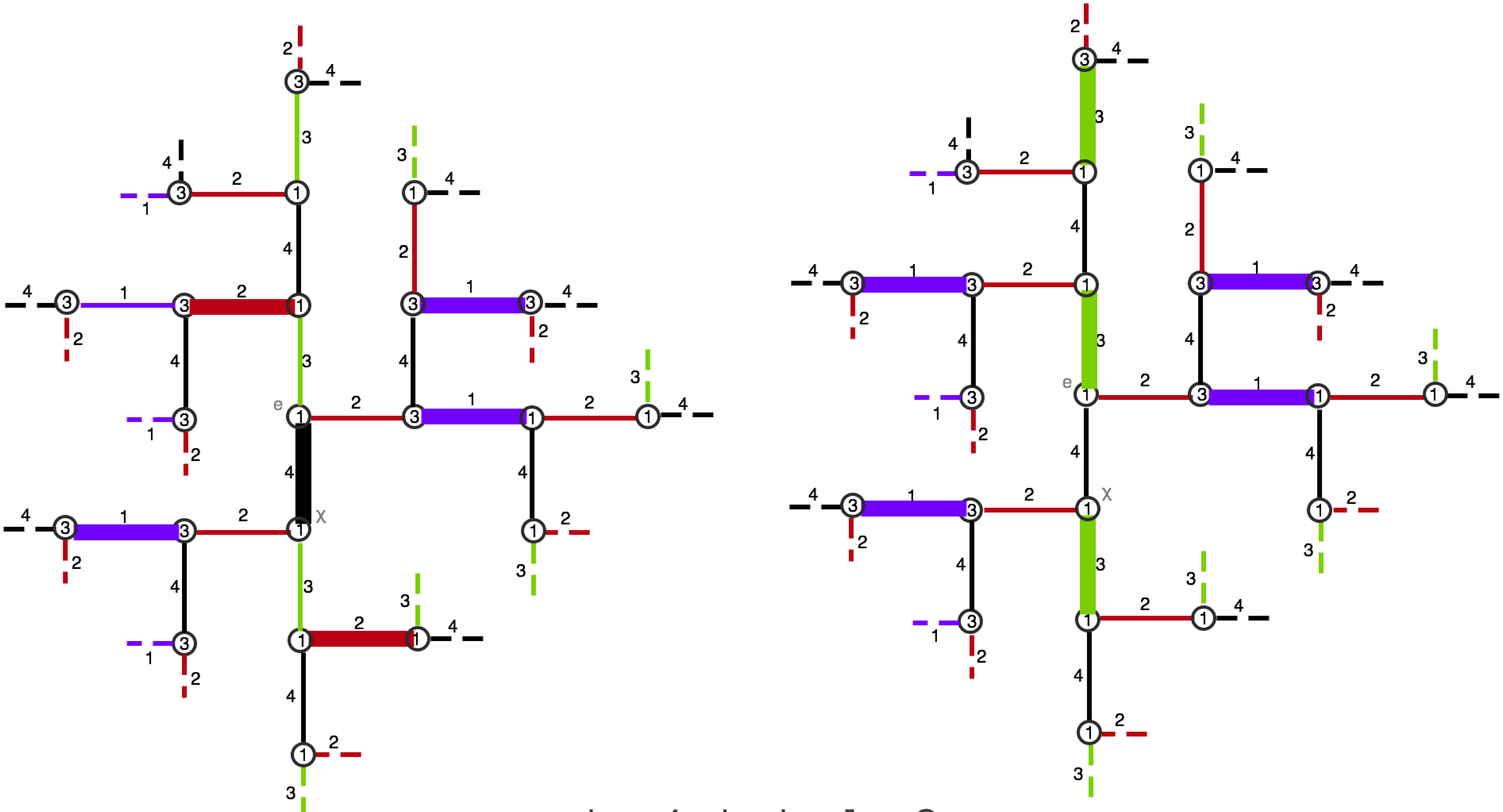
$\Omega(k - 1)$





Incompatible outputs

this work
$\Omega(k-1)$

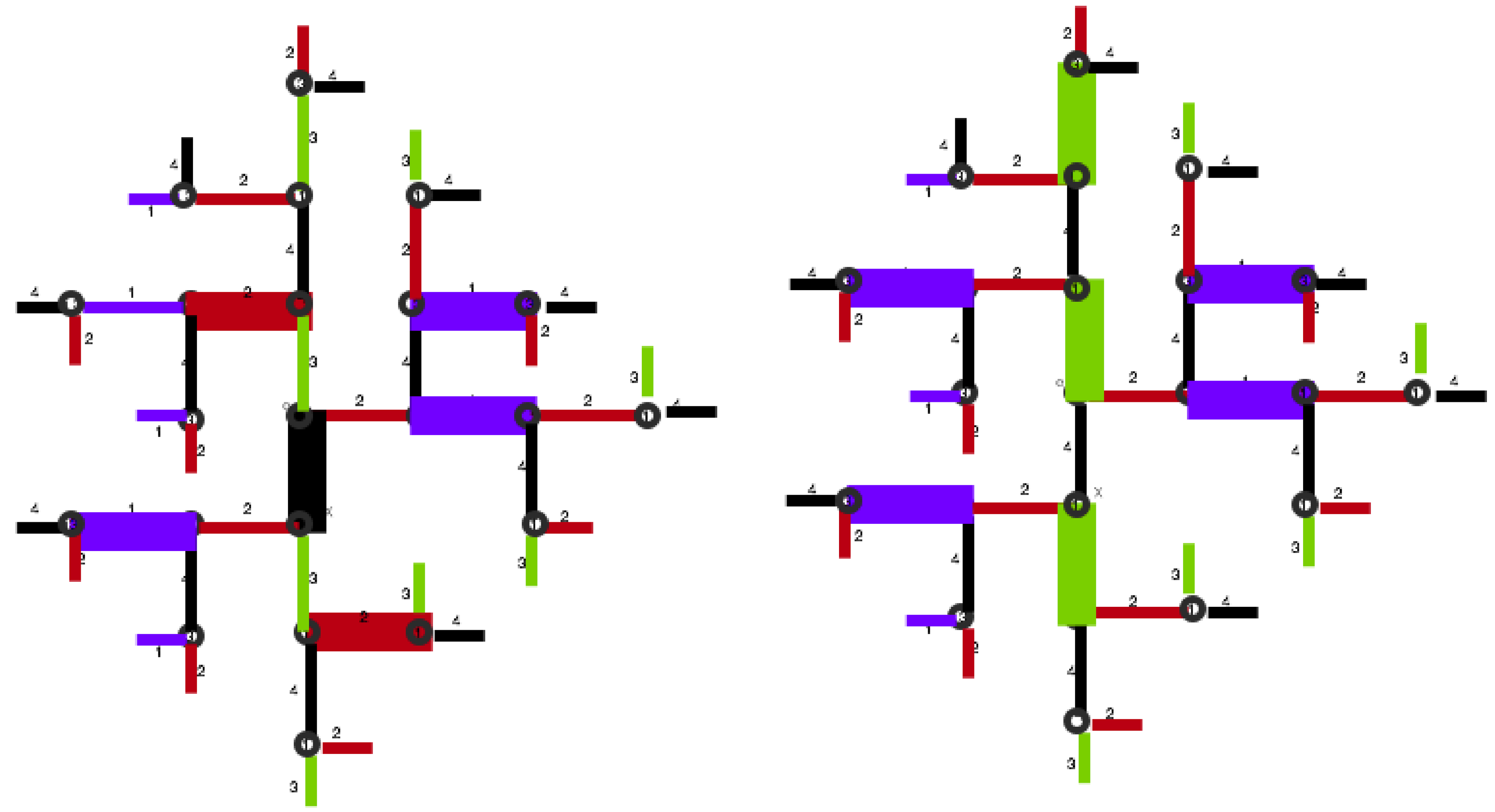


k = 4, d = k - 1 = 3
same radius-2 view

Induction

this work

$\Omega(k - 1)$

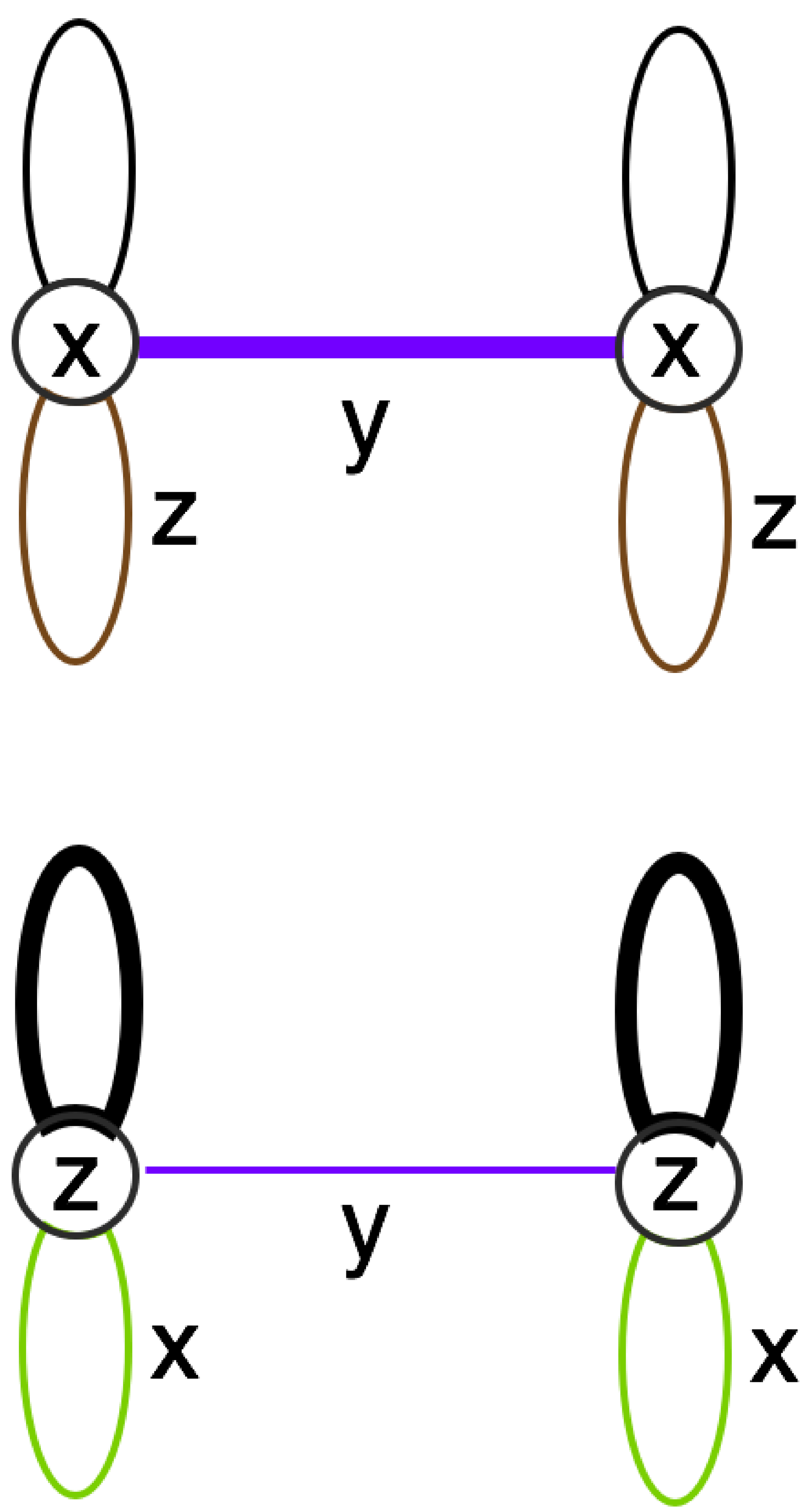


Two degree-i templates such that a root node
- produces different outputs;
- radius-(i - 1) neighbourhoods are identical

i = 1: base case
i > 1: by induction
i = d = k - 1: result

Base case

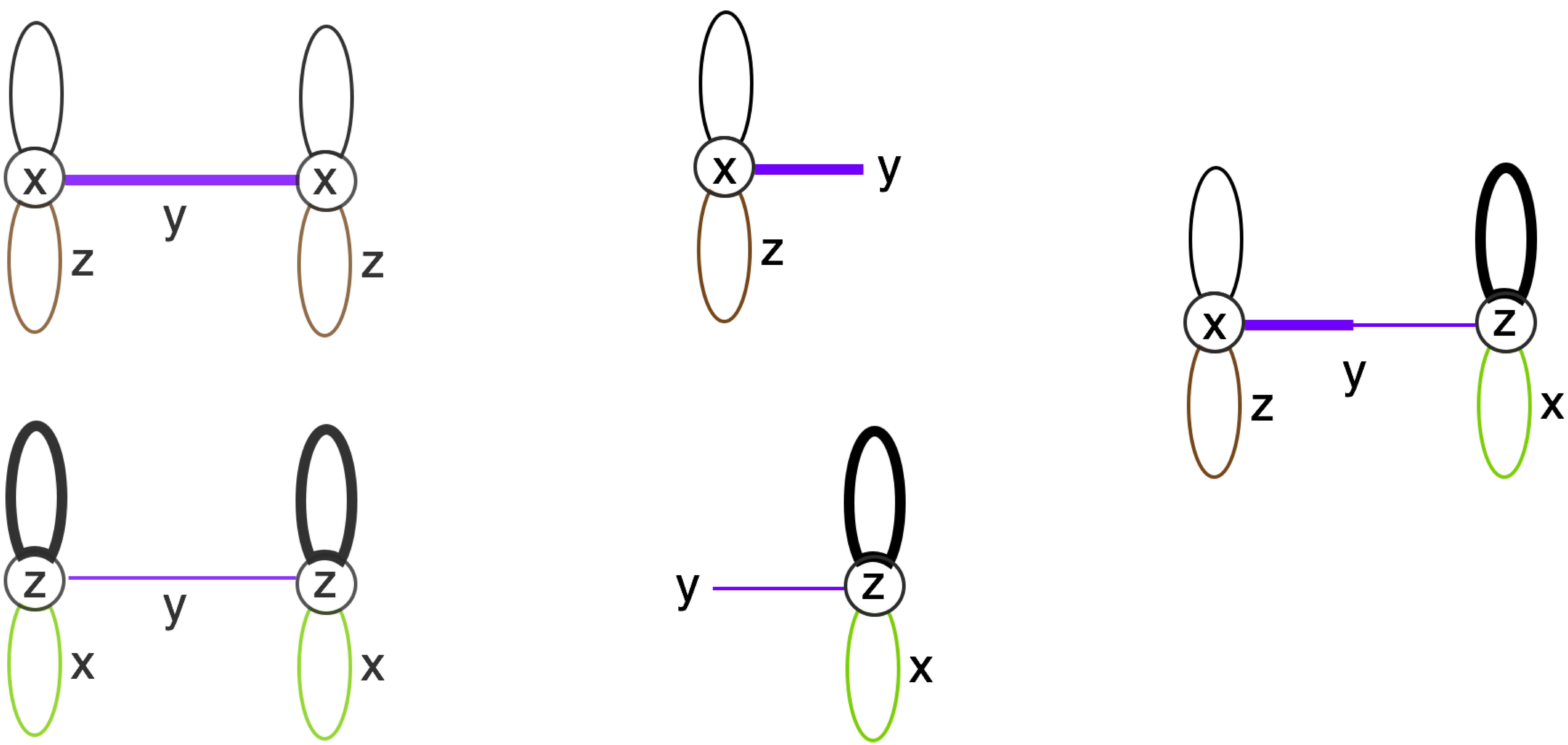
this work
$\Omega(k - 1)$



Base case

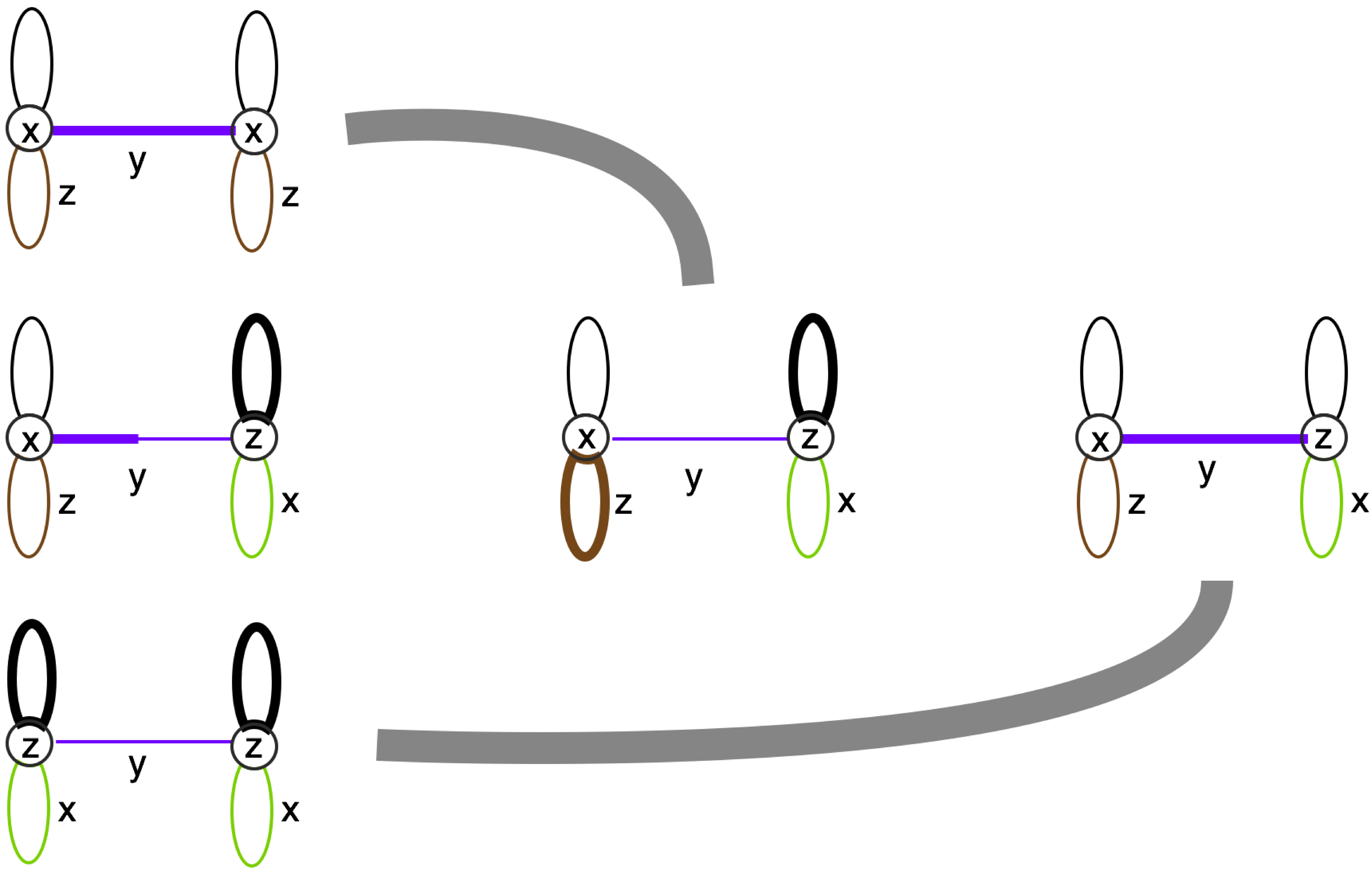
this work

$\Omega(k-1)$



Base case

this work
$\Omega(k-1)$

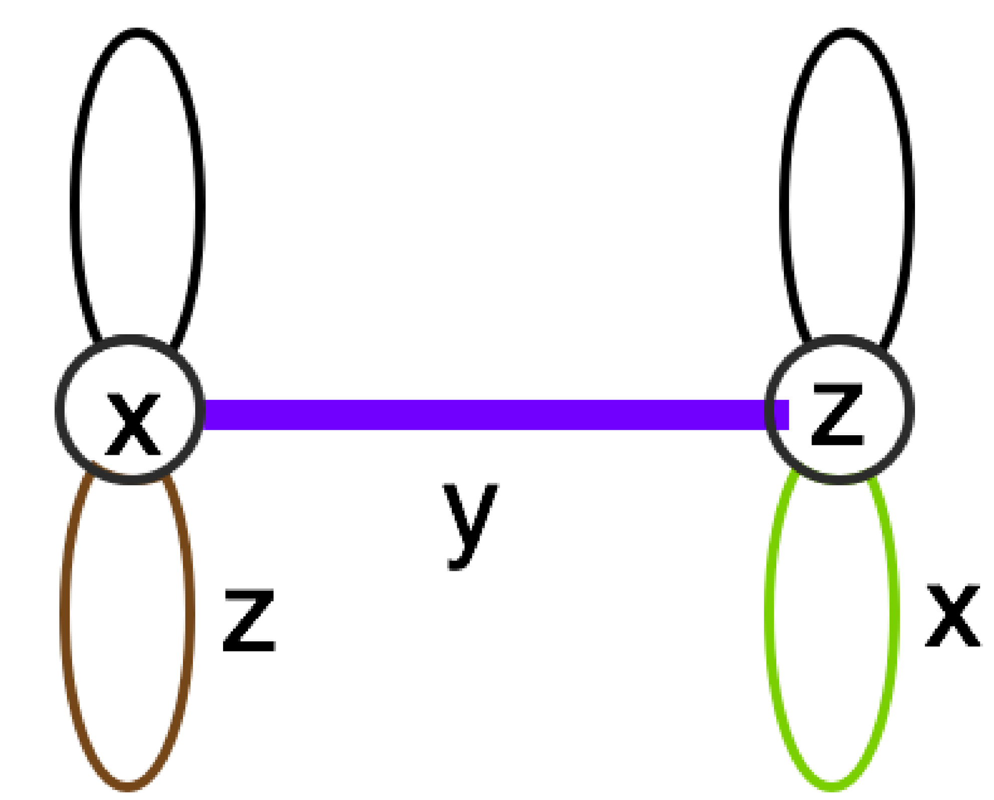


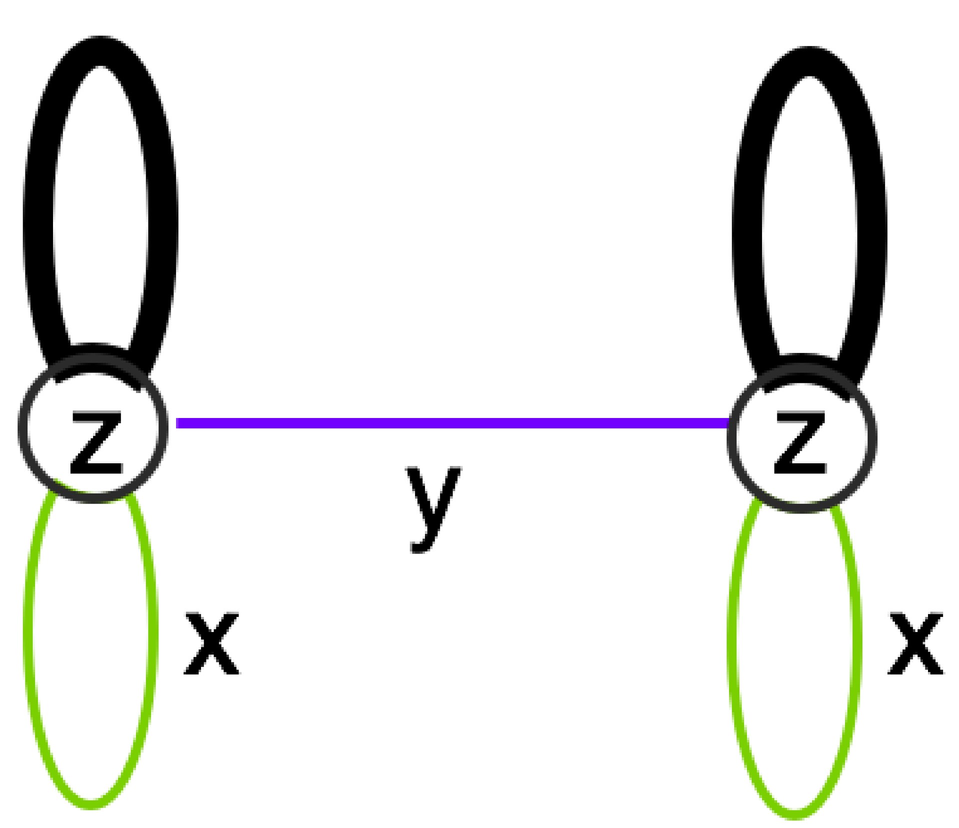
degree 1 templates, same radius-0 view, different output

Base case

this work

$\Omega(k-1)$

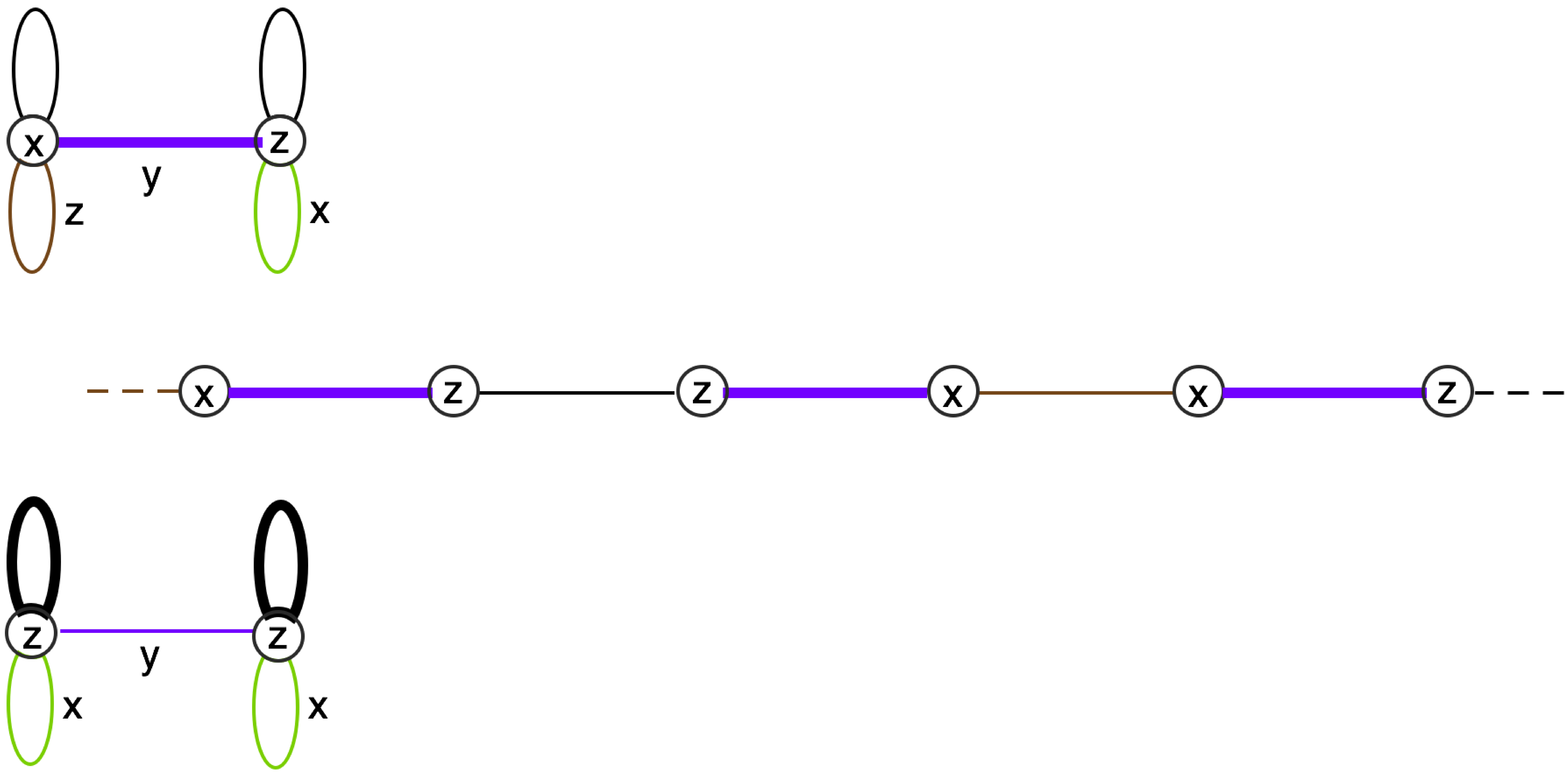




Inductive step

this work

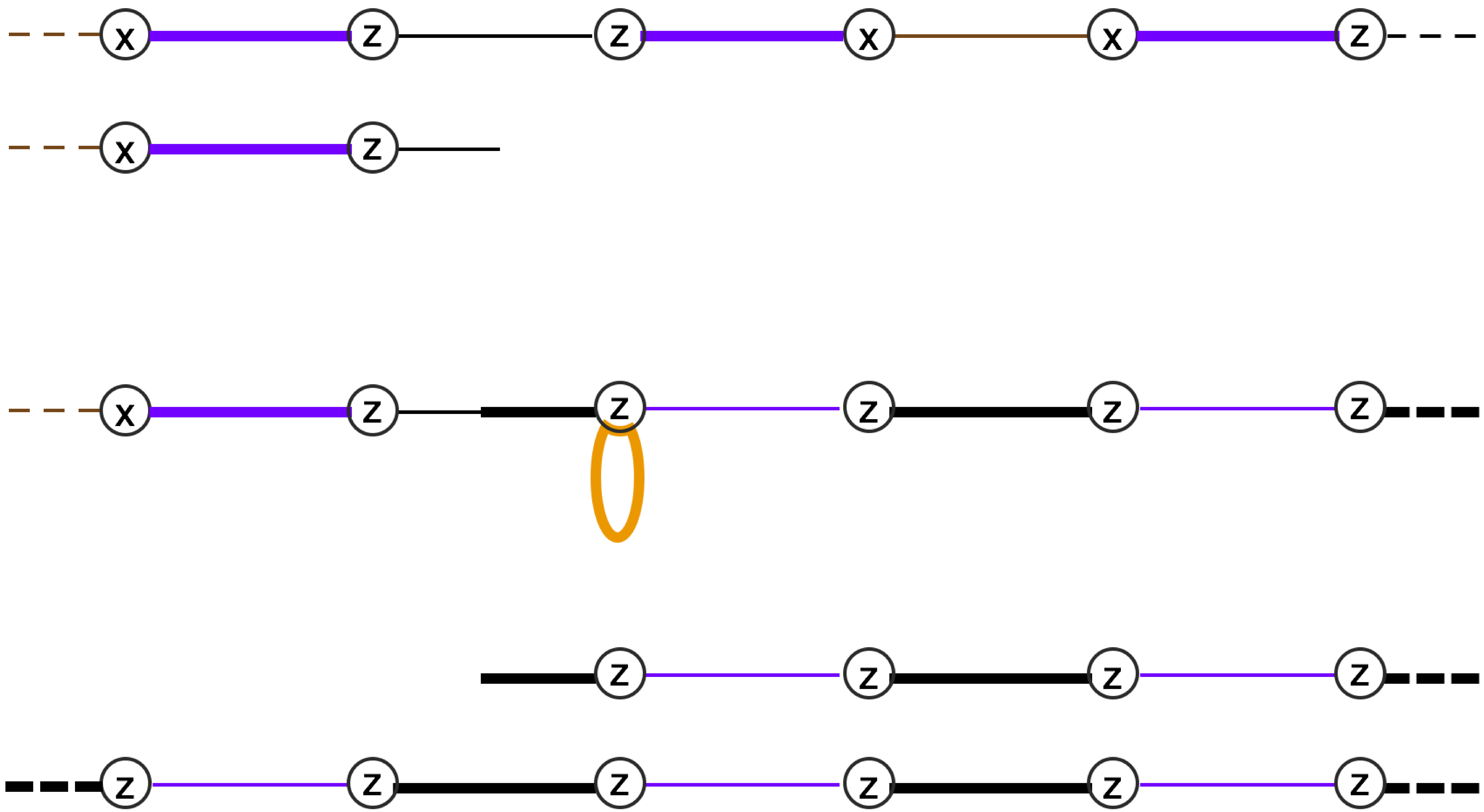
$\Omega(k-1)$



Inductive step

this work

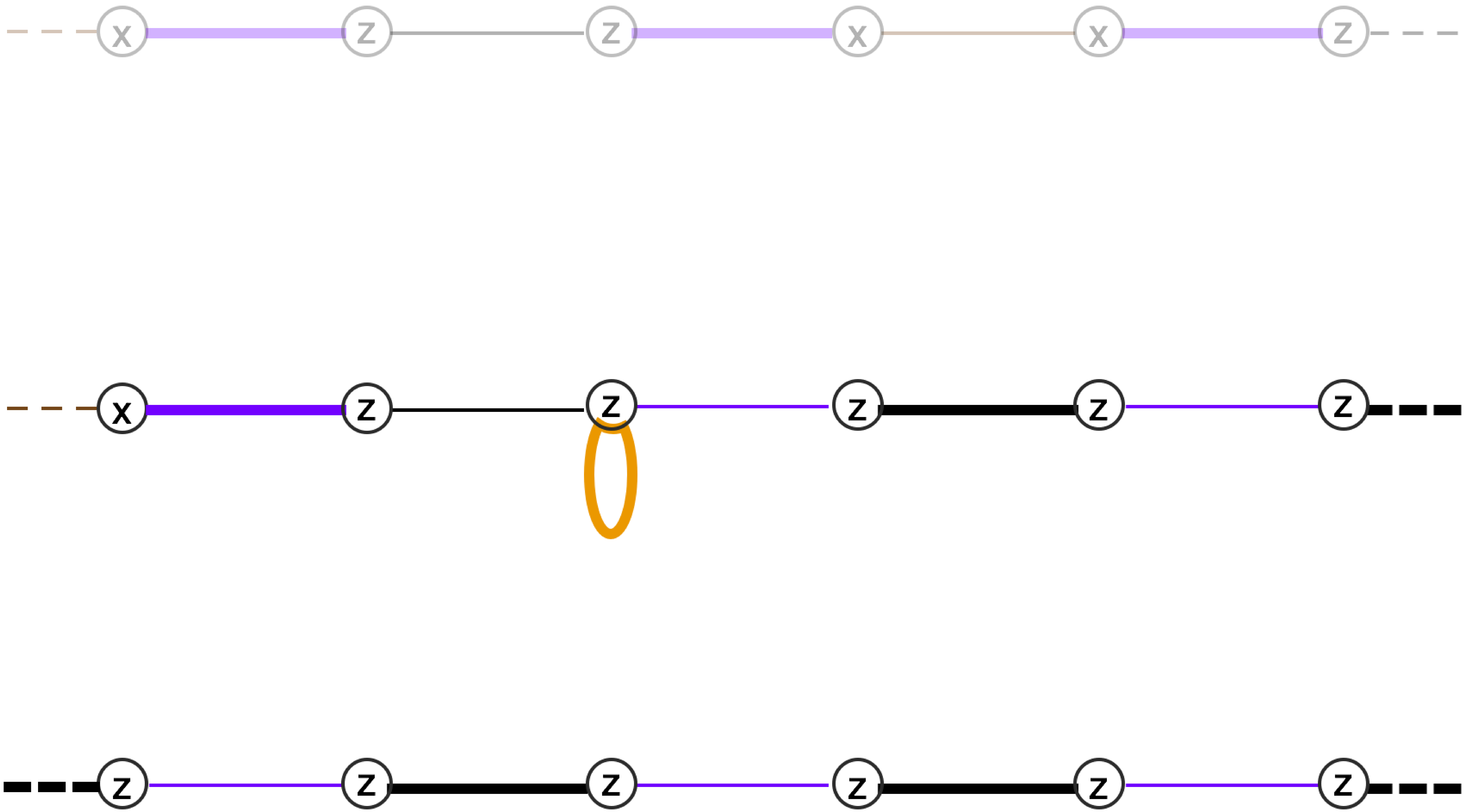
$\Omega(k-1)$



Inductive step

this work

$\Omega(k-1)$

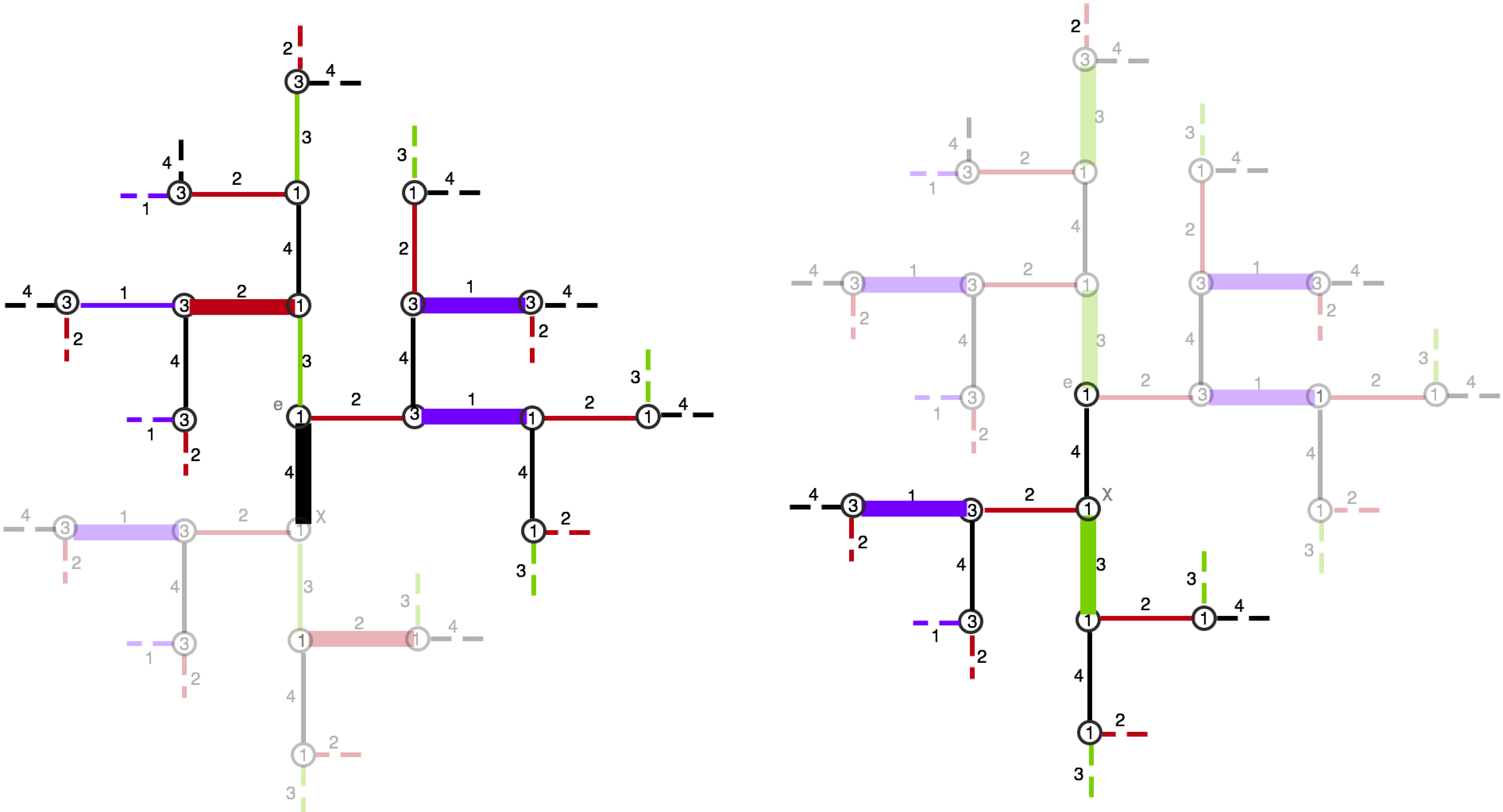


degree-2 templates, same radius-1 view, different output

degree-3

this work

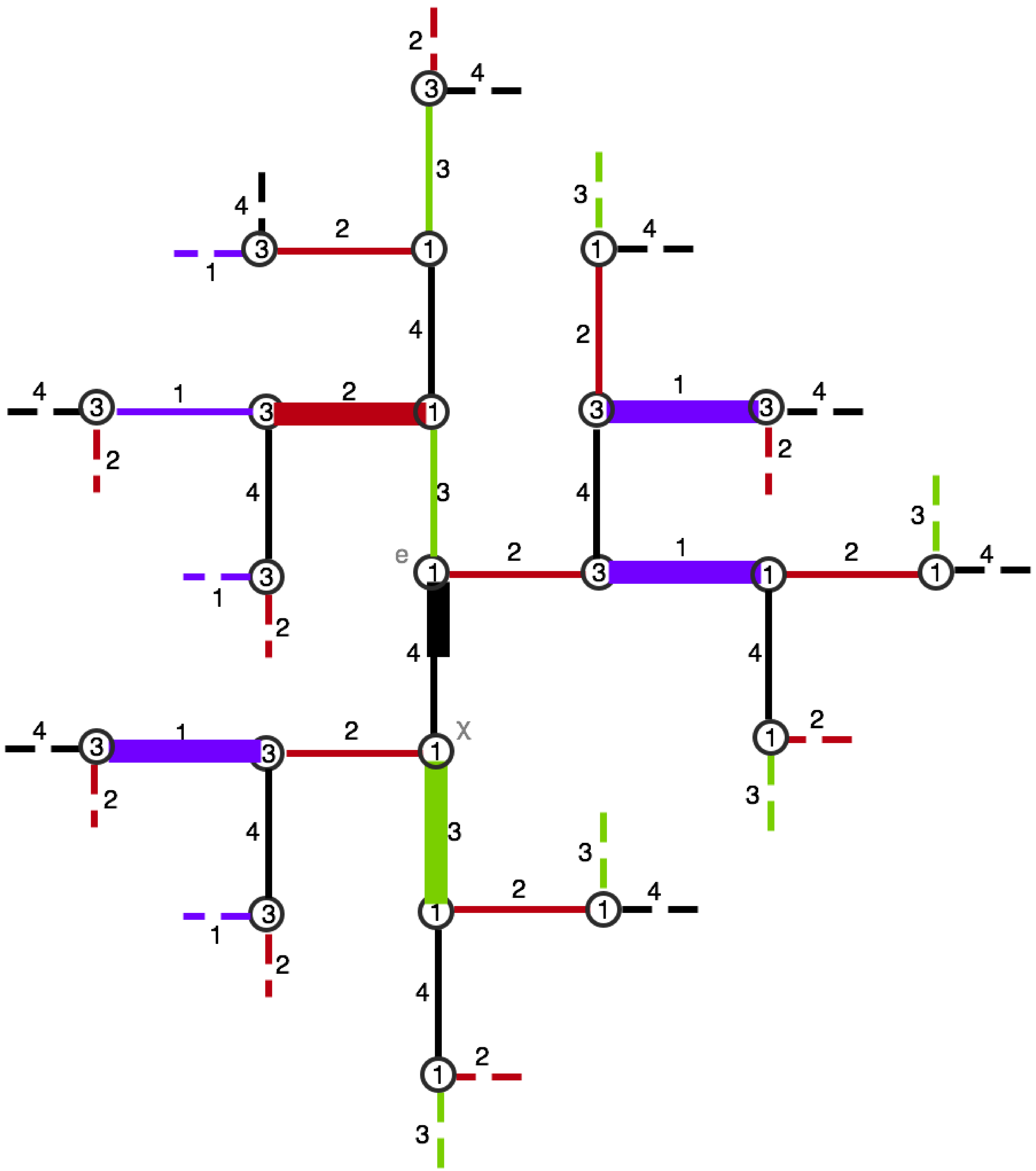
$\Omega(k-1)$

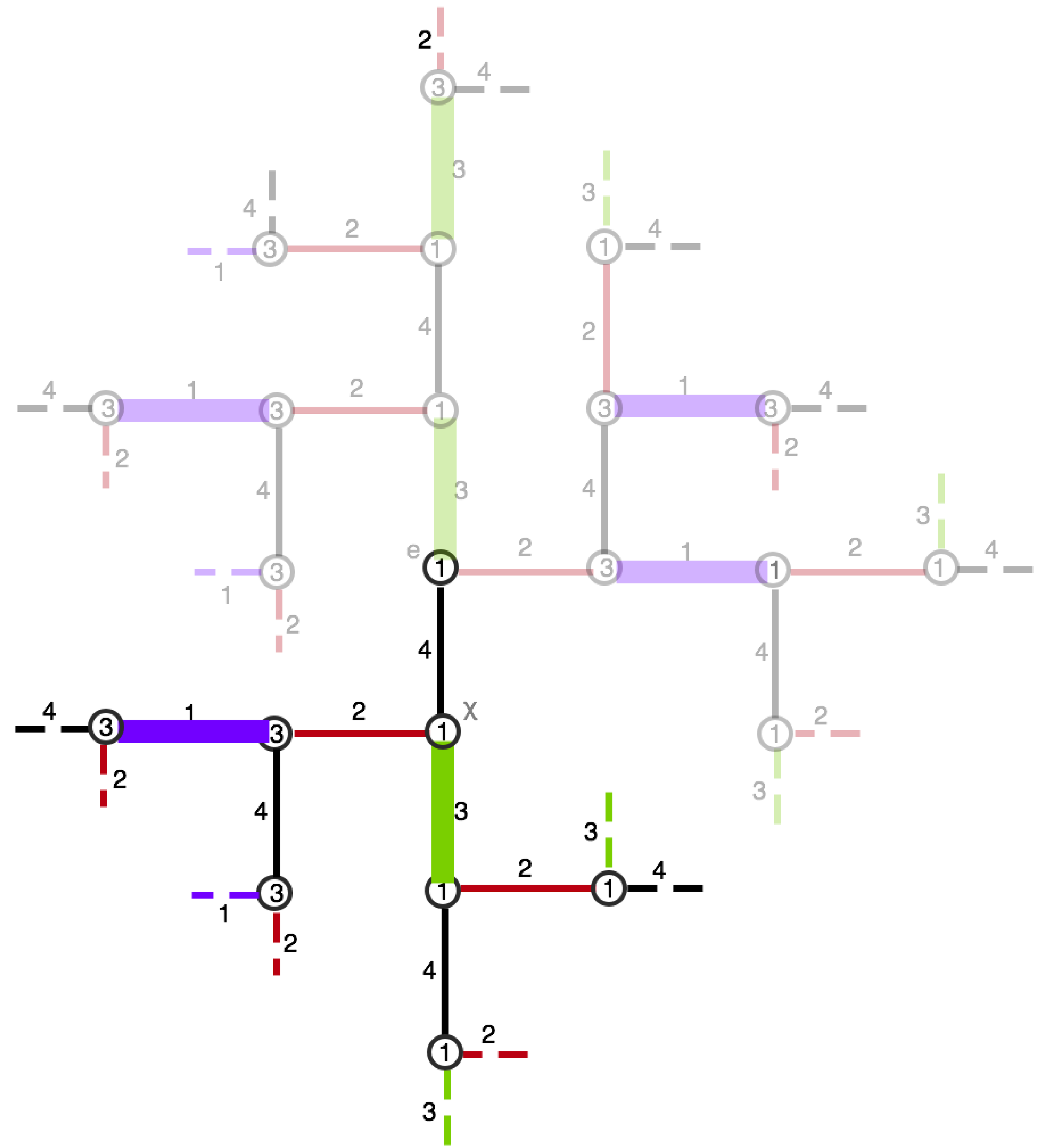


degree-3

this work

$\Omega(k - 1)$

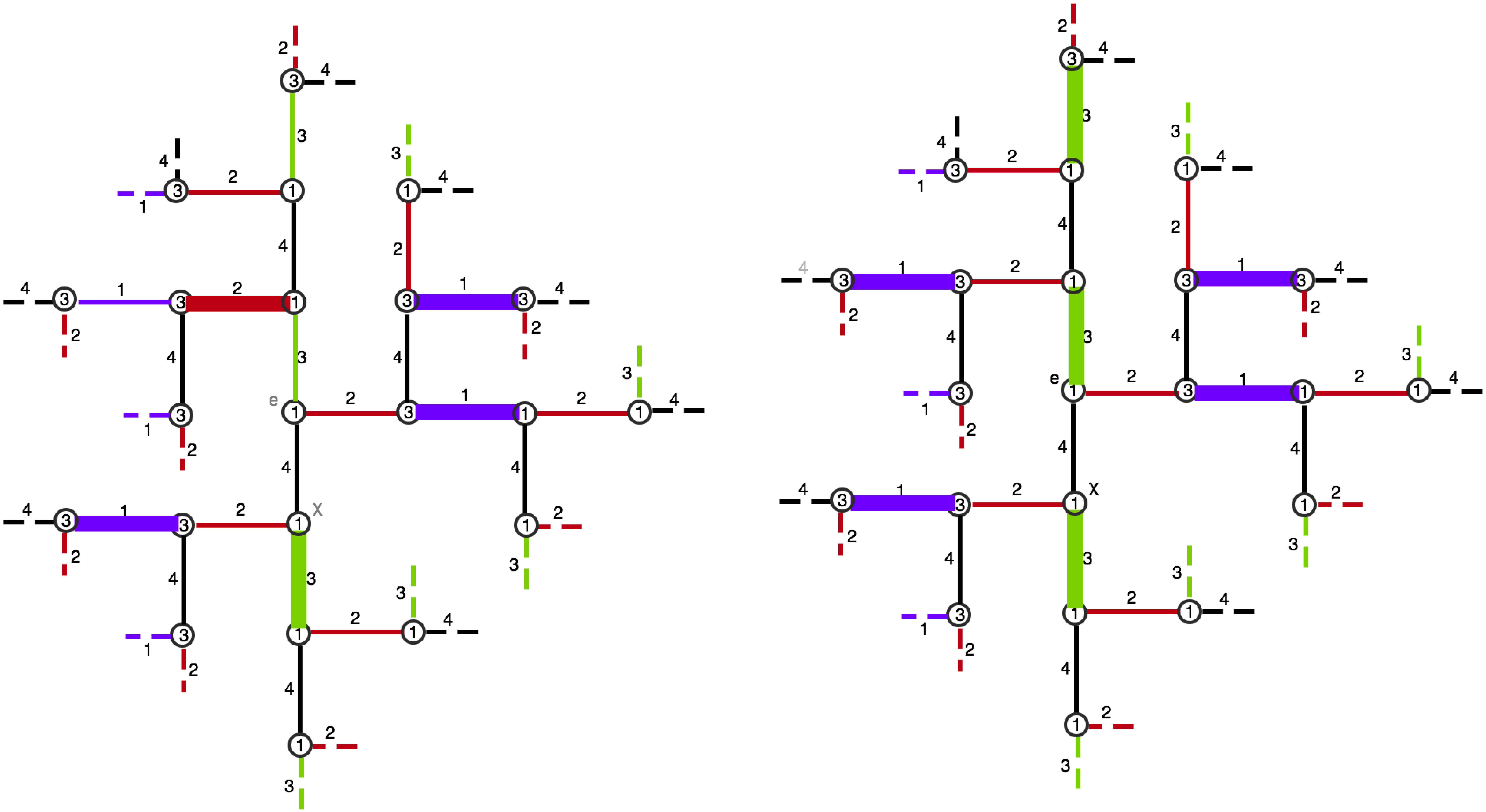




degree-3

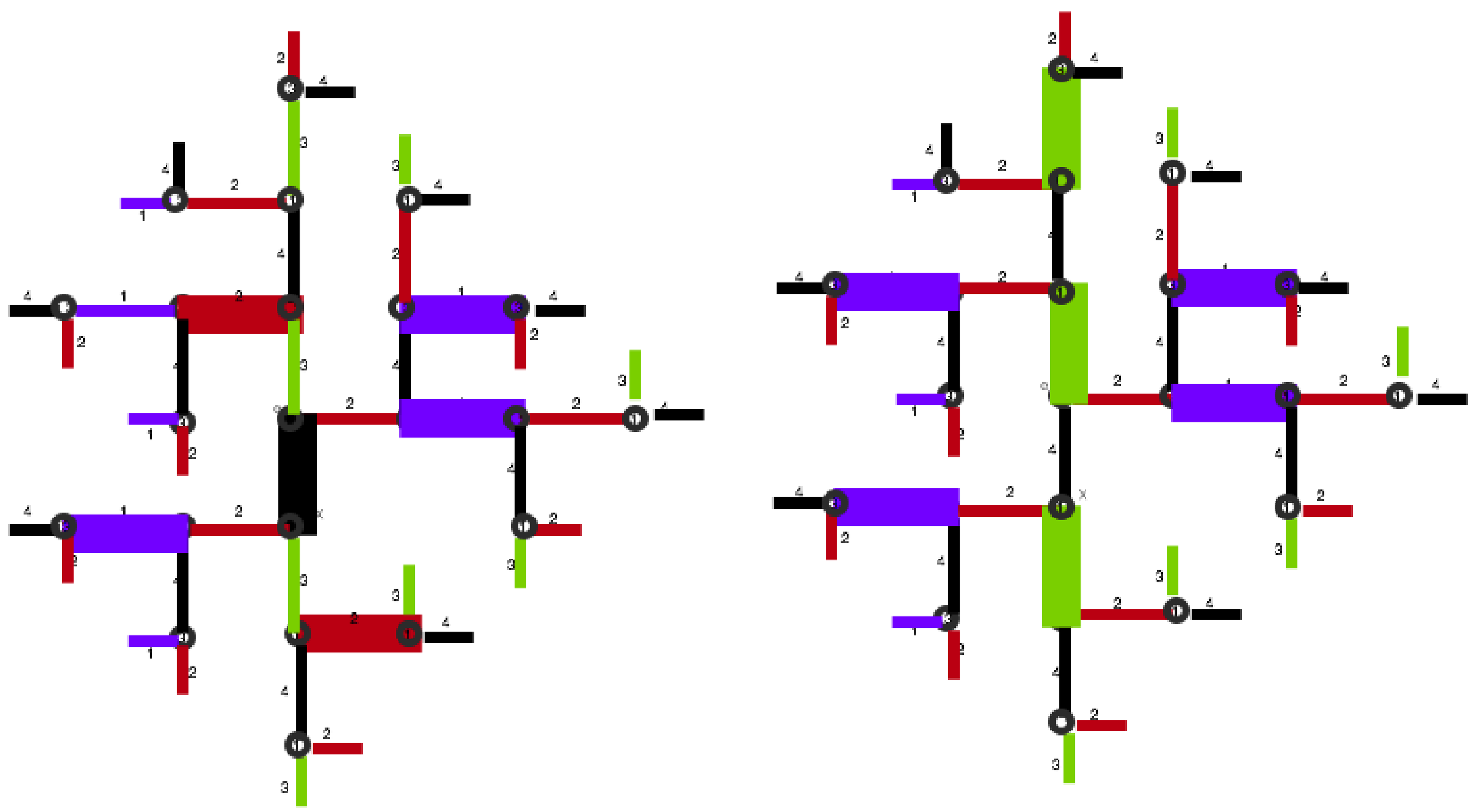
this work

$\Omega(k - 1)$



degree-2 templates, same radius-2 view, different output

Theorem 2



Let **k >= 3** and **d = k - 1**

Assume a distributed algorithm that finds a maximal matching in any d-regular k-colored graph.
Then there are two d-regular k-colored graphs A, B such that a node u_e has the same (d - 1)-radius view in A and B and u_e is unmatched in A and matched in B

Theorem 2

Let **k >= 3** and **d = k - 1**

Assume a distributed algorithm that finds a maximal matching in any d-regular k-colored graph.
Then there are two d-regular k-colored graphs A, B such that a node u_e has the same (d - 1)-radius view in A and B and u_e is unmatched in A and matched in B

$\therefore$

Theorem 1

$$\Omega(k-1) \overset{\Delta \leq k}{\Rightarrow} \Omega(\Delta)$$

Let k be a positive integer. A deterministic distributed algorithm that finds a maximal matching in any anonymous, k-edge-colored graph requires at least k - 1 communication rounds

anonymous,
k-edge-colored

$O(\Delta + \log^* k)$

thight bound for distributed maximal matching
in anonymous, k-edge-colored graphs

this work

$\Omega(\Delta)$

previous work

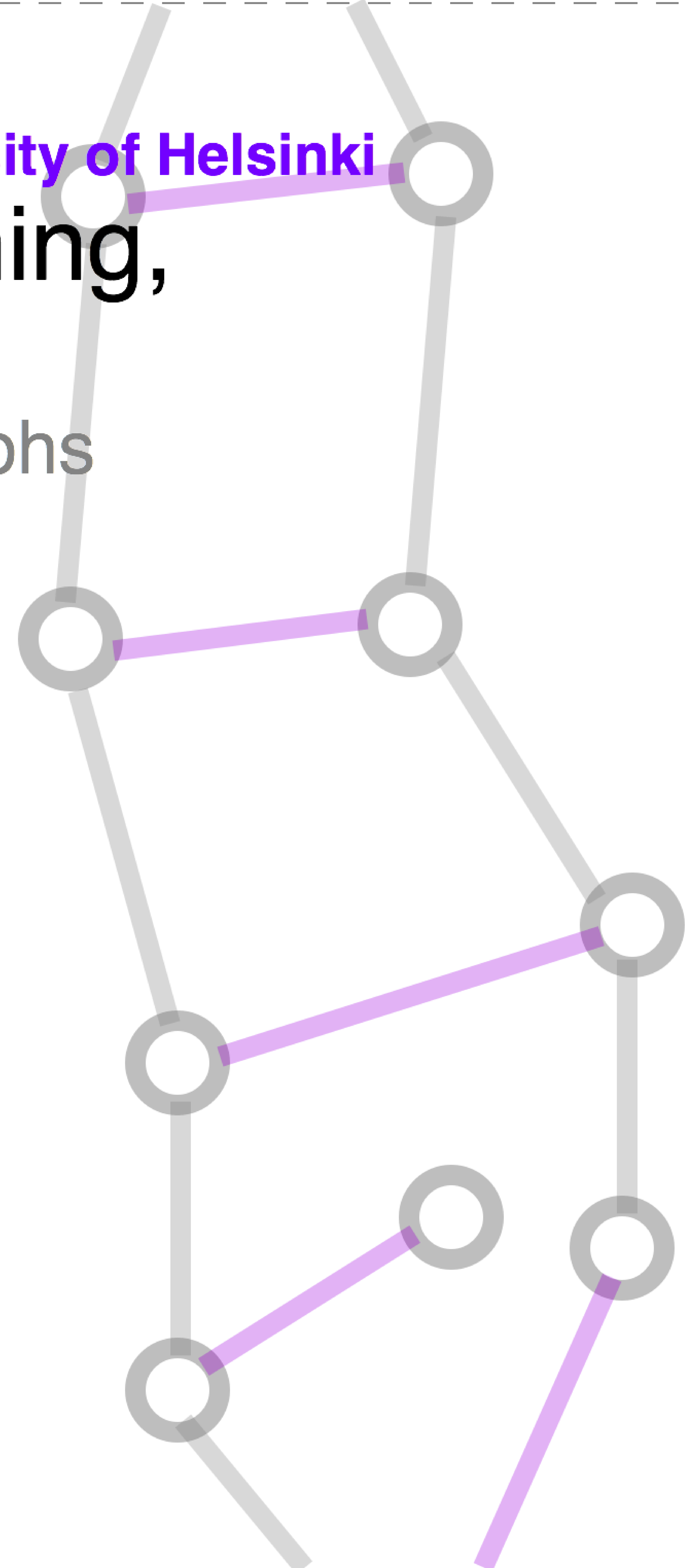
$\Omega(\log^* k)$

$\therefore \quad \Omega(\Delta + \log^* k)$

Juho Hirvonen and Jukka Suomela, University of Helsinki

Distributed maximal matching, Greedy is optimal

in anonymous, k-edge-colored graphs



$$\Theta(\,\Delta + \log^* k\,)$$

Andrea Lattuada