Performance-Effective and Low-Complexity Task Scheduling for Heterogeneous Systems

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Outline

1. Task Scheduling
    - Classic Model
    - Theoritical Background
    - Heterogeneity
    - Algorithms
2. HEFT & CPOP
    - Heterogeneous Earliest Finish Time (HEFT)
    - Critical-Path-on-Processor (CPOP)
    - Experiments
3. Conclusion

Basics

- **Static task scheduling**.

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  - **Output**: schedule with minimal overall completion time

Tasks

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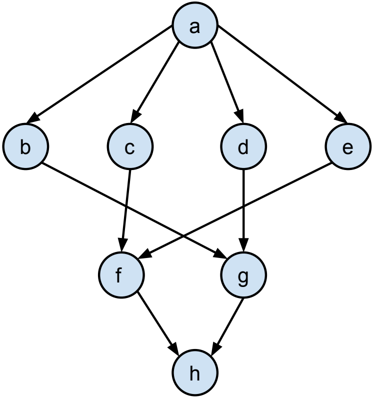
- $G = (V, E)$



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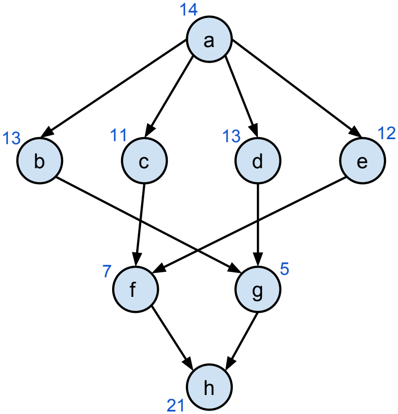

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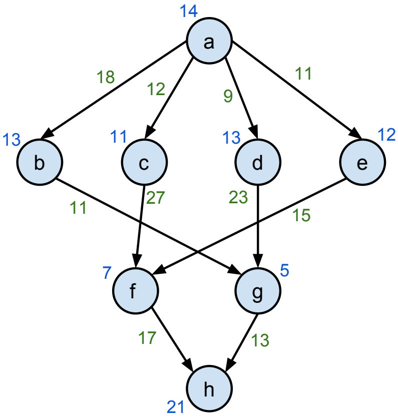

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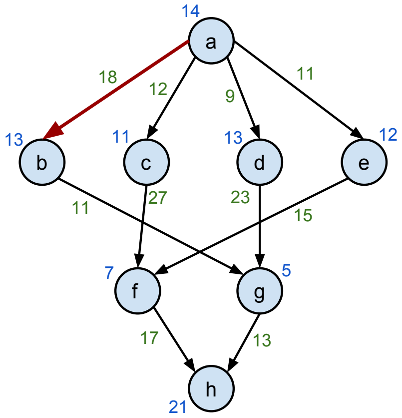

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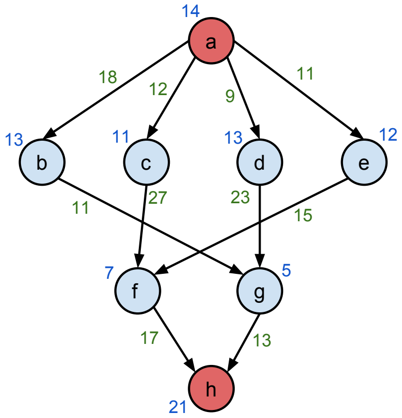

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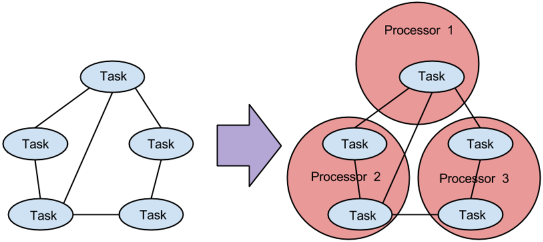

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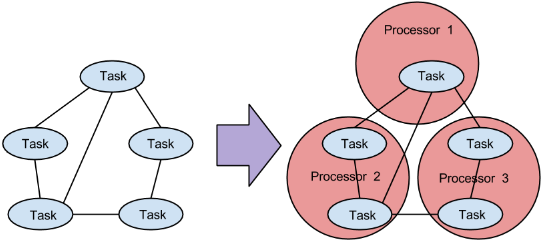

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3. By reducing 3-PARTITION in polynomial time to SCHED, it's shown that SCHED is strongly NP-hard

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- Proved by Garey & Johnson, 1975

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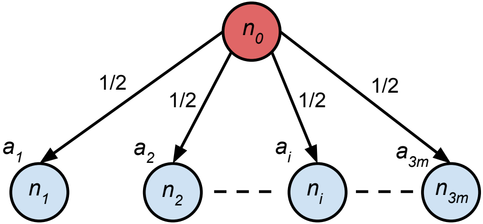

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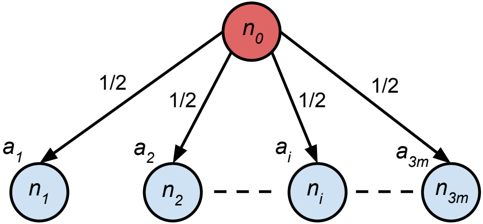

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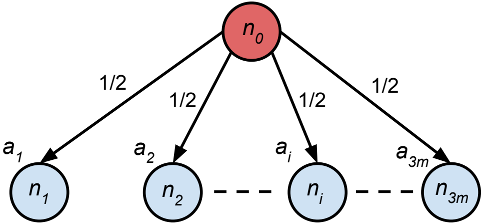

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- $|\mathbf{V}| = 3m + 1$ nodes, $|P| = m$ and $T = B + 1.5$

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- 3-PARTITON reduces to SCHED $\Rightarrow$ SCHED is strongly NP-hard

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SCHED$(G, \mathbf{P}_\infty)$ is NP-complete

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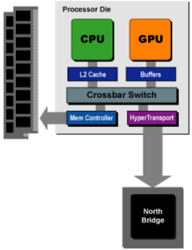

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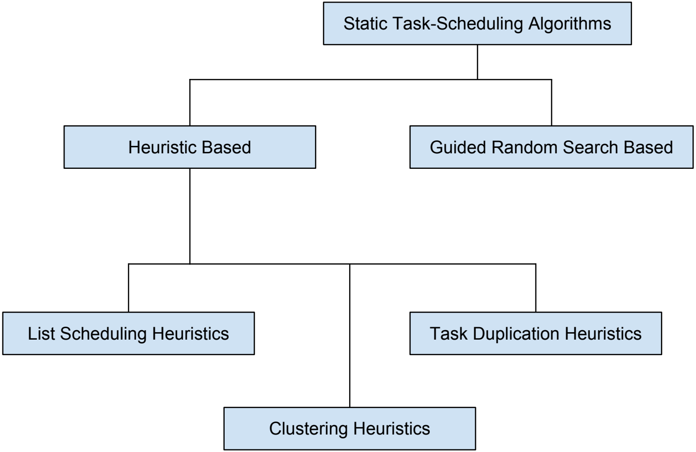
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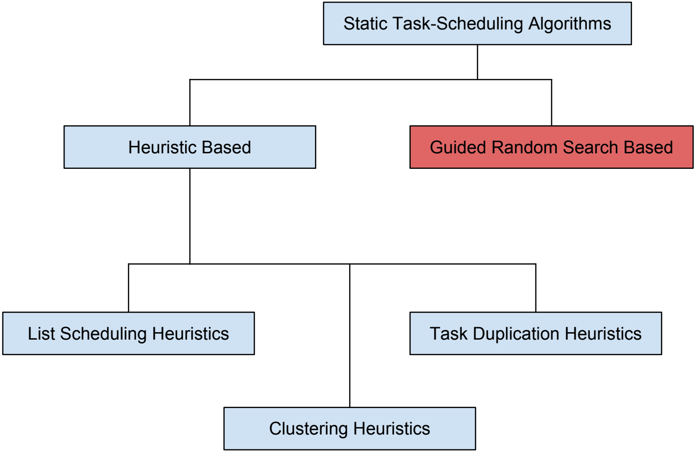
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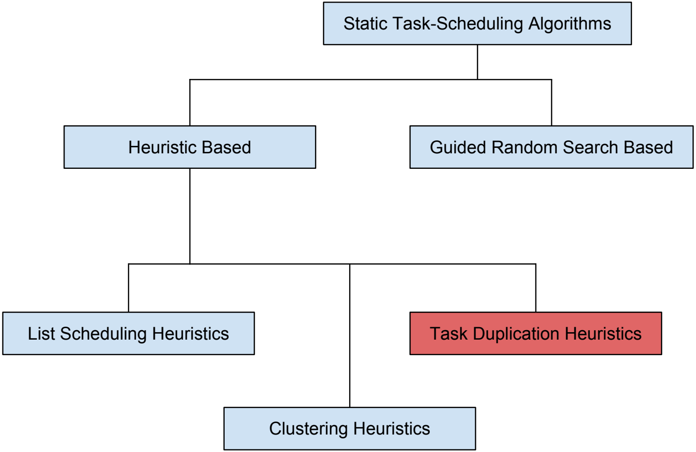
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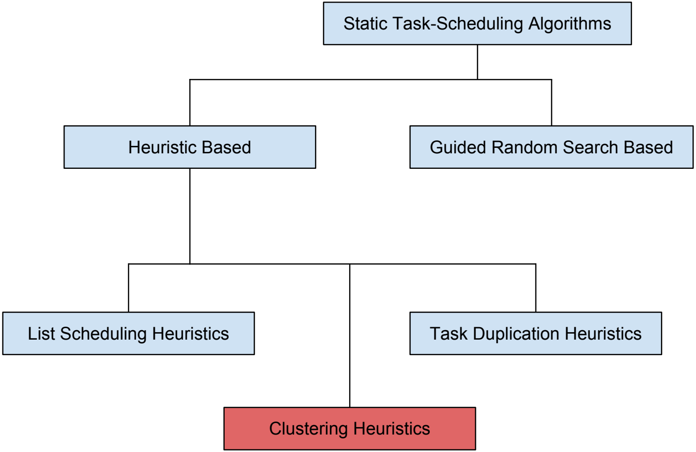
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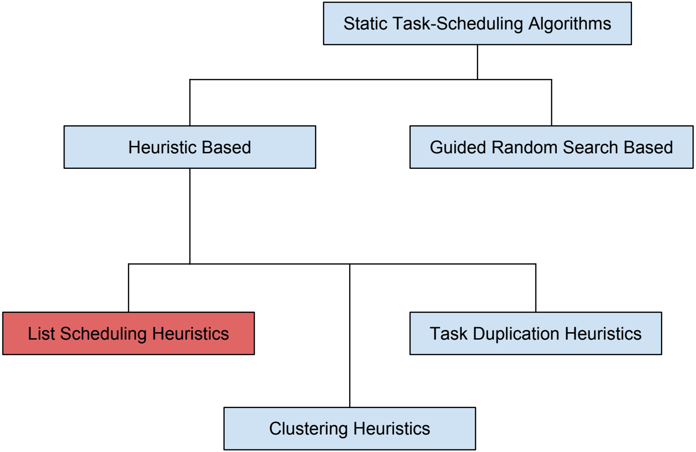

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- Complexity dependent on scheme in phases

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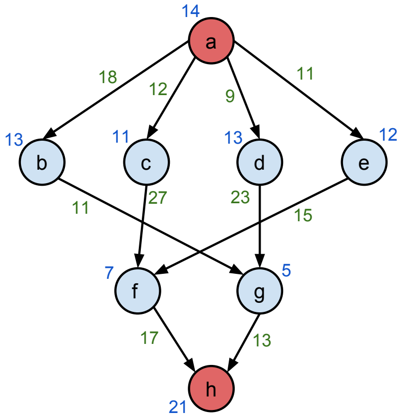

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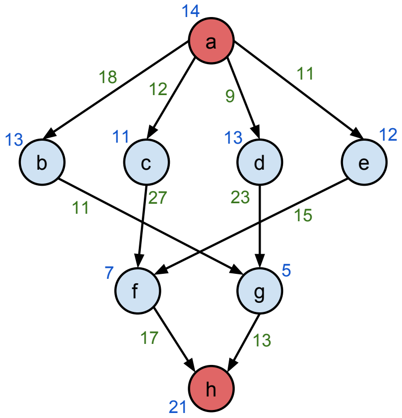
- $\mathbf{rank_d}$
- Cost up to task
- Defined recursively



HEFT & CPOP

- Implement list-scheduling heuristics
- HEFT
  - Heterogeneous Earliest Finish Time
  - Implements an insertion-based policy
- CPOP
  - Critical-Path-on-Processor
  - Tries to speed up the execution of tasks on the critical path

HEFT

- 2 phases

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  - task prioritization

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- Processor selection:
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  - Assign task to processor which minimizes EFT

CPOP

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- Implemented using a priority queue

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- Both cases consider an insertion-based scheduling policy

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  - Molecular Dynamics Code

Competing Algorithms

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Competing Algorithms

- Dynamic-Level Scheduling (DLS)
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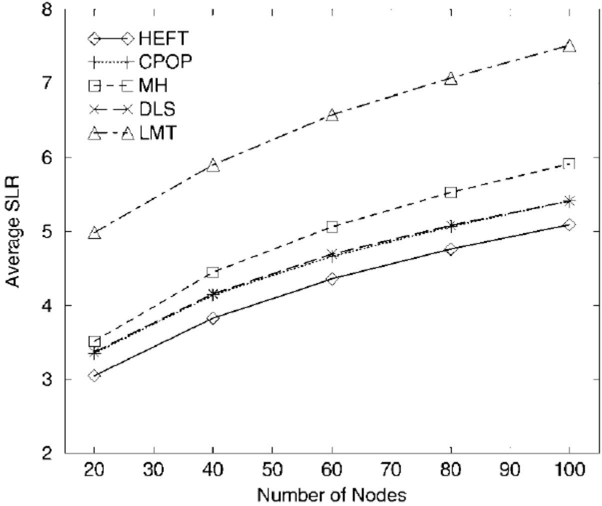
Comparison Metrics

- **Schedule Length Ratio(SLR)**
  - SLR is a normalized schedule length for an algorithm
  - The SLR value for an algorithm is given by:

$$SLR = \frac{makespan}{\sum_{n_i \in CP_{min}} \min_{p_j \in Q} w_{ij}}$$

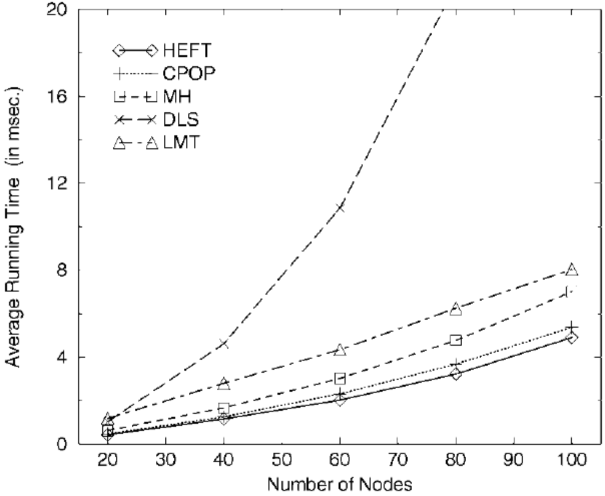
- **Run time**

Avg. SLR



HEFT
CPOP
MH
DLS
LMT
Average SLR
Number of Nodes
8
7
6
5
4
3
2
20
30
40
50
60
70
80
90
100

Avg. Runtime



HEFT
CPOP
MH
DLS
LMT
Average Running Time (in msec.)
0
4
8
12
16
20
20
30
40
50
60
70
80
90
100
Number of Nodes

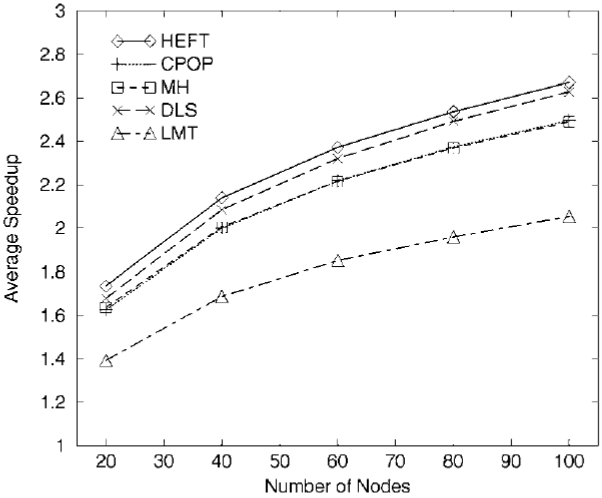
Comparison Metrics (contd.)

- **Speedup**
  - The speedup value for a given graph is computed by dividing the sequential execution time by the parallel execution time
  - It's value is given by:

$$Speedup = \frac{\sum_{n_i \in CP_{min}} \min_{p_j \in Q} w_{ij}}{makespan}$$

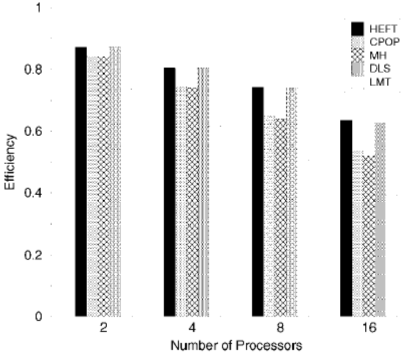
- **Efficiency**
  - Efficiency is calculated by dividing the speedup by the number of processors

Avg. Speedup



HEFT
CPOP
MH
DLS
LMT
Average Speedup
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
20
30
40
50
60
70
80
90
100
Number of Nodes

Efficiency - Gaussian Elimination



Result Summary

- HEFT pwns everyone
- CPOP isn't far behind
- Alternative task prioritizing
- and processor selection policies for HEFT

Conclusion

- Static TS is NP-complete in a strong sense
- Heterogeneous systems are important, TS on them more so
- Two list heuristic based algorithms: CPOP and HEFT
- Significantly outperform their competitors

Questions?

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