Scalable Rational Secret Sharing

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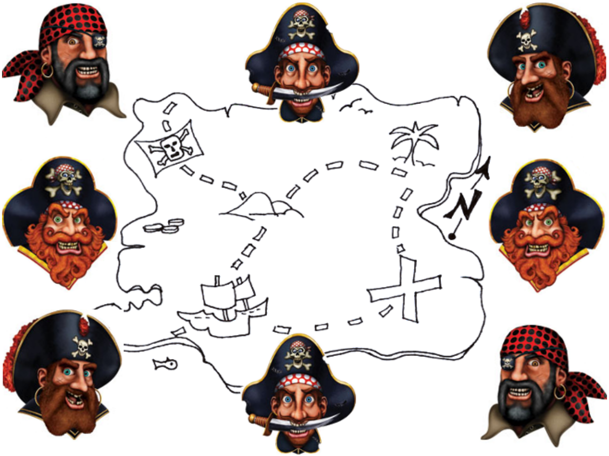
Starting Example



Starting Example

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- The shares are distributed among the players.
- Each share on its own should not reveal the secret.
- Combining all the shares reconstructs the secret.
- The players are selfish and rational.
- Each player prefers to
  1. learn the secret by him self
  2. learn the secret together with other
  3. not learn the secret at all.

Starting Example

Secure Secret Sharing

A **secure secret sharing scheme** distributes shares so that anyone with fewer than n shares has no additional information about the secret than someone with no shares at all.

Classical secure secret sharing Scheme

Shamir's Scheme: A polynomial of degree $n-1$ can be reconstructed using n points.

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- Encode the secret as the first coefficient of a random polynomial f of degree $n-1$
- The shares are points $(i, f(i))$ on the polynomial
- Any n players can reconstruct the polynomial using interpolation

The Problem with Selfish Players

A selfish player will never send his share to the other players, as he has to fear, that he will not get the other players shares back.

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$\Rightarrow$ **We need to adapt the protocol.**

Solution

Our Goals

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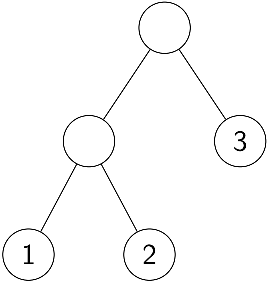
- No player can improve substantially by deviating from the protocol.
- The protocol is scalable:
  - message complexity per agent: $O(1)$
  - time complexity: $O(\log |agents|)$

Outline

- Dealer's Protocol
  - is only active at the beginning
  - prepares the input for the player's protocol
- Player's Protocol
  - is played in rounds
  - in round X the secret is revealed

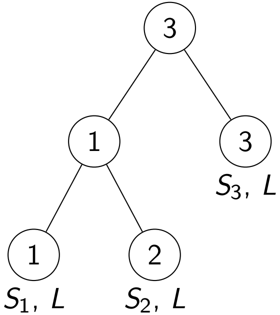
Dealer's Protocols

Dealer's Protocol



- assign all players to leaves

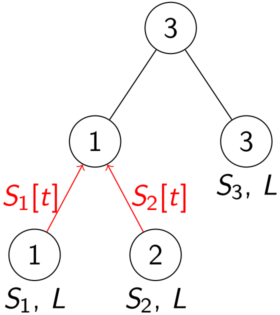
Dealer's Protocol



- assign all except for one player to remaining nodes
- give each player his *share list* S_i and a *list of potential secrets* L

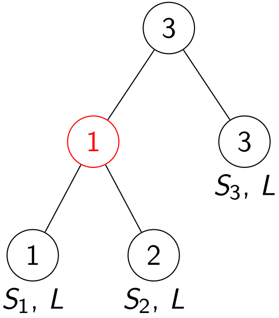
Player's Protocols

Player's Protocol – Up-Stage



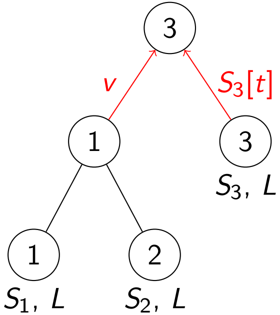
- children send their shares to parent

Player's Protocol – Up-Stage



- verify $S_1[t]$
- verify $S_2[t]$
- construct new share v

Player's Protocol – Up-Stage



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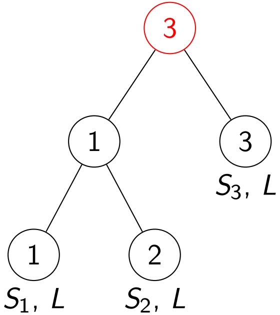
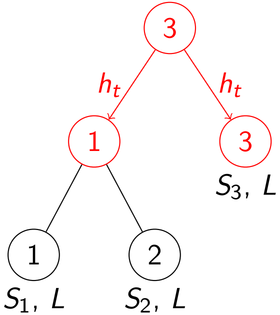


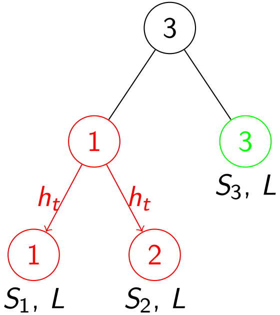
- verify v
- verify $S_3[t]$
- construct h_t

Player's Protocol – Down-Stage



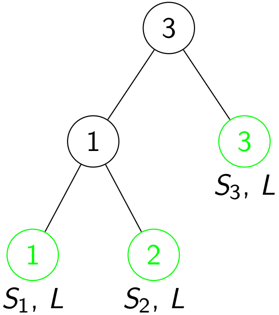
- send h_t to children

Player's Protocol – Down-Stage



- verify h_t
- send h_t to children

Player's Protocol



- verify h_t
- all players know h_t
- if $h_t = 0$ then $t = X$

Analysis

Truthfulness

There are only two ways in which a player can deviate from the protocol:

1. Send fake messages
2. Leave the protocol before the secret was revealed

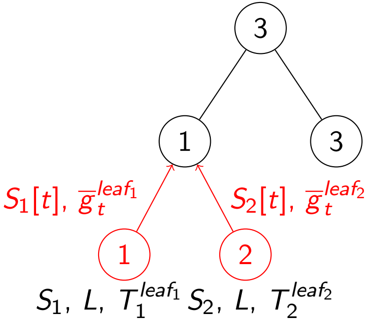
Verification by Tag Values

- Dealer prepares
  - T_i^w: a tag list for the sending node w
  - $H_j^{w',w}$: a list of verification tokens for the receiving node w'
- Node w sends a tag $\overline{g}$ from T_i^w along with its share v.
- Node w' asserts

$$c = a \cdot v + b \cdot \overline{g}$$

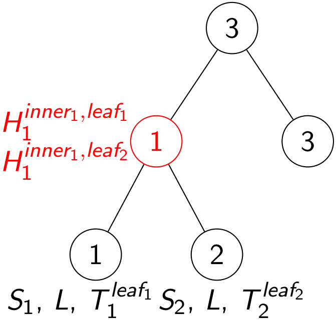
  where $(a, b, c) \in H_j^{w',w}$.
- All entries in the lists S, T and H are elements taken from a finite field F_q.

Verification by Tag Values



- children send their shares and tags to parent

Verification by Tag Values



- verify that $c_t^{leaf_i} = a_t^{leaf_i} \cdot S_i[t] + b_t^{leaf_i} \cdot \overline{g}_t^{leaf_i}$
- construct new share v

Send a Fake Message

Proposition (3.1)

The probability that a faked message will satisfy the verification function is $\frac{1}{q-1}$.

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- Send a fake message v' with the corresponding g' (not known by the sender).

Exit Protocol

If at any time during a round t of the player's protocol some player i catches some other player cheating, i outputs the current secret $L[t]$ and leaves the protocol.

Leave the Protocol

- Don't transmit the fact $h_t = 0$.
- Guess the real secret with a sufficiently high probability.

Leave the Protocol

Lemma (3.2)

A player deviating from the protocol cannot increase his expected payoff by more than ϵ unless his probability of successfully learning the secret by deviating is at least $p = \frac{(U - U_- + \epsilon)}{(U_+ - U_-)}$.

Lemma (3.3)

A player who initially received a list of length α has at most $\frac{1}{\alpha - t}$ chance of (correctly) guessing the position of the secret on round t if it has not already been revealed.

Ensuring Trustfulness

We need $p \geq \frac{(U - U_- + \epsilon)}{(U_+ - U_-)}$ for cheating to be profitable and we know that $p = \frac{1}{\alpha - t}$:

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$$\alpha - t \leq \frac{(U_{+} - U_{-})}{(U - U_{-} + \epsilon)}$$

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$\Rightarrow$ Choose padding $Y \geq \frac{(U_+ - U_-)}{(U - U_- + \epsilon)}$ to ensure truthfulness.

Proof of Efficiency

- The expected number of messages sent by each player is $O(1)$.
- The expected number of bits sent is $O(log(q))$.
- The expected overall latency is $O(log(n))$.

Conclusions

Summary

- An algorithm for rational secret sharing, where no player can improve substantially by deviating from the protocol.
- The mechanism is scalable in terms of latency and number of messages sent by each player.

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	Scalable RSS	**Previous Work**
messages per player per round	$O(1)$	$O(n)$
latency per round	$O(log\ n)$	$O(n)$
E[# rounds]	$O(1)$	$O(n)$

Questions?