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Distributed Computing

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Principles of Distributed Computing
Exercise 13: Sample Solution

1 Determining the Median

As stated in the hint, we start with initializing the nodes to give them IDs from $1, \ldots, n$. Now the node with ID i transmits its token in time slot i. Each node uses two variables to count the number of tokens which were transmitted with a higher or lower number. After the execution each node knows how many tokens were larger/smaller than its own. Thus, the node whose token is the median can simply transmit it afterwards.

Let us briefly analyze the time and space used by this. We know from the lecture that initialization takes $\mathcal{O}(n)$ rounds. The sending of each token afterwards takes exactly n rounds, i.e., does not increase the asymptotic runtime.

Each node needs to store a new ID of up to n, i.e., needs $\mathcal{O}(\log n)$ space. The variables from the algorithm can also be stored by using $\mathcal{O}(\log n)$. After the initialization, we require that each node keeps two counters to count how many numbers were larger/smaller than our own. But since we do not need to store the values, this requires only $\log n$ space.

The correctness follows by the construction of the algorithm. The unique node whose upper and lower counter has the same value, can broadcast it and thus all nodes are aware of it.