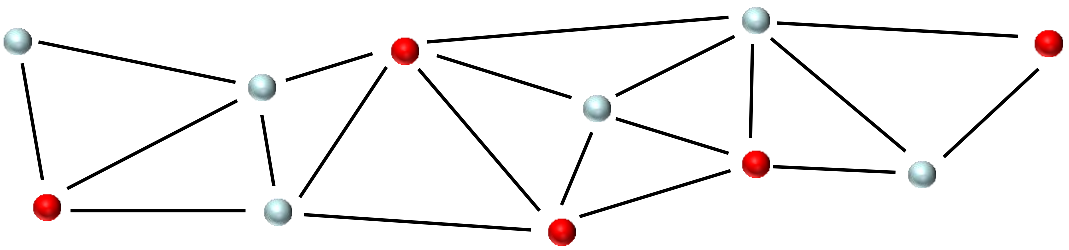
Lower Bound Example: Minimum Dominating Set (MDS)

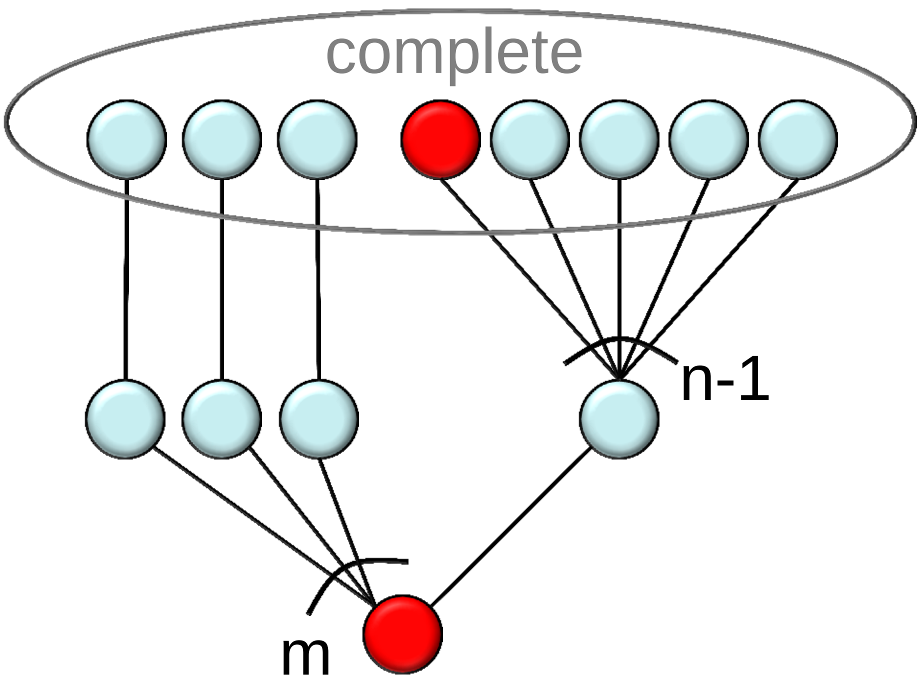
- Input: Given a graph (network), nodes with unique IDs.
- Output: Find a Minimum Dominating Set (MDS)
  - Set of nodes, each node is either in the set itself, or has neighbor in set



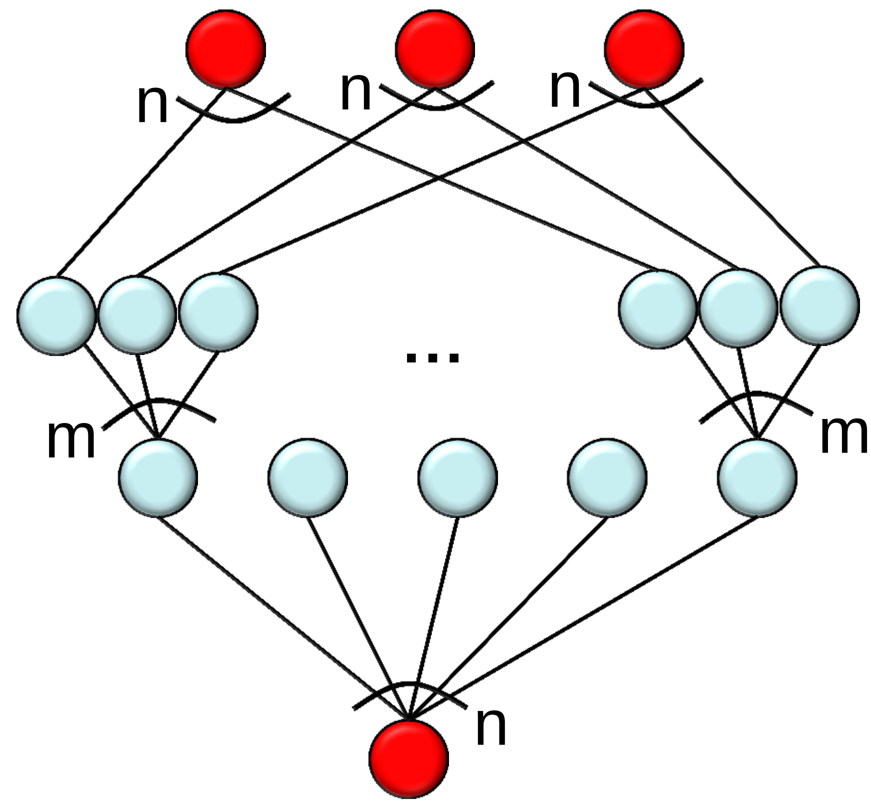
- Differences between MIS and MDS
  - Central (non-local) algorithms: MIS is trivial, whereas MDS is NP-hard
  - Instead: Find an MDS that is “close” to minimum (approximation)
  - Trade-off between time complexity and approximation ratio

Lower Bound for MDS: Intuition

- Two graphs (m << n). Optimal dominating sets are marked red.



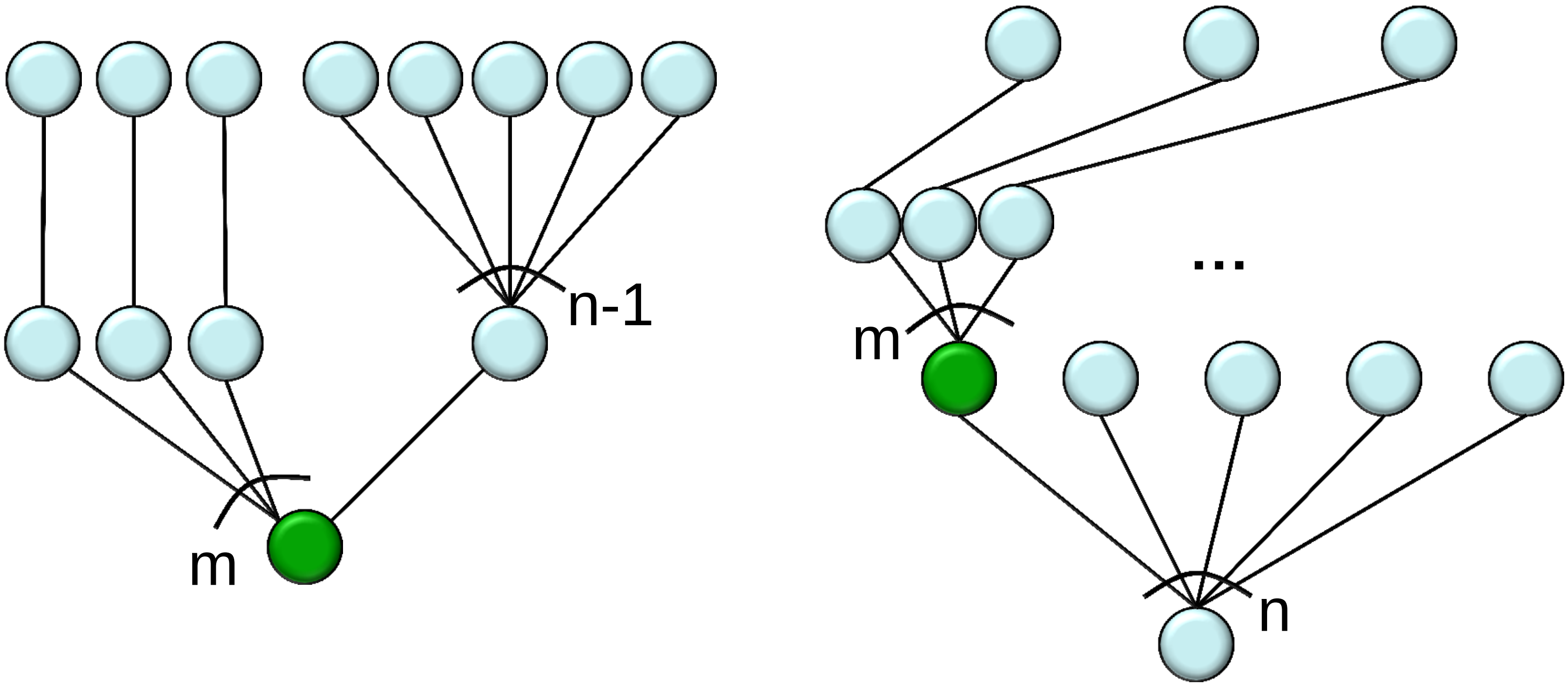
$|DS_{OPT}| = 2.$



$|DS_{OPT}| = m+1.$

Lower Bound for MDS: Intuition (2)

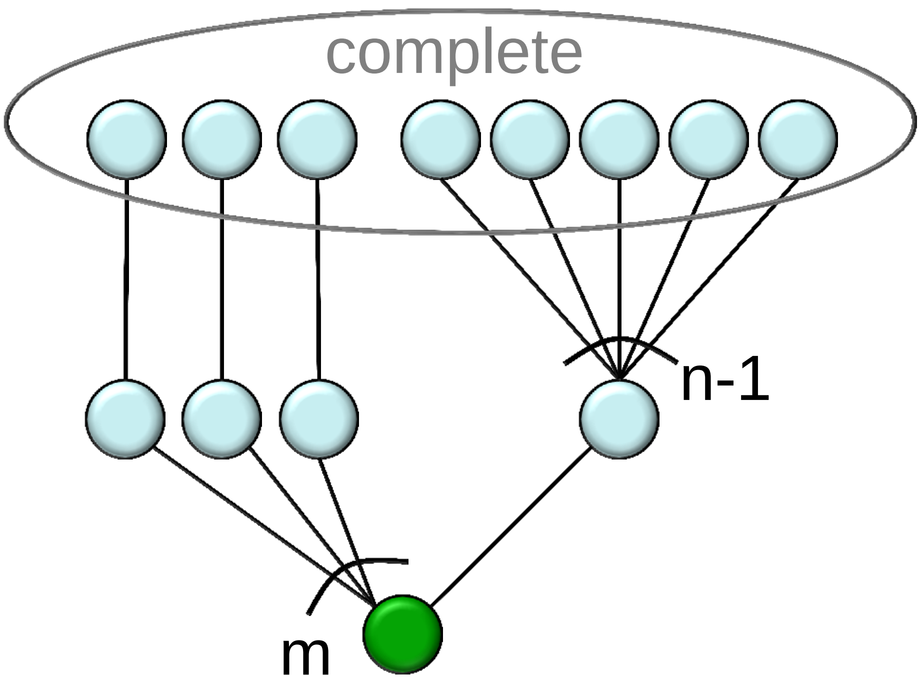
- In local algorithms, nodes must decide only using local knowledge.
- In the example green nodes see exactly the same neighborhood.

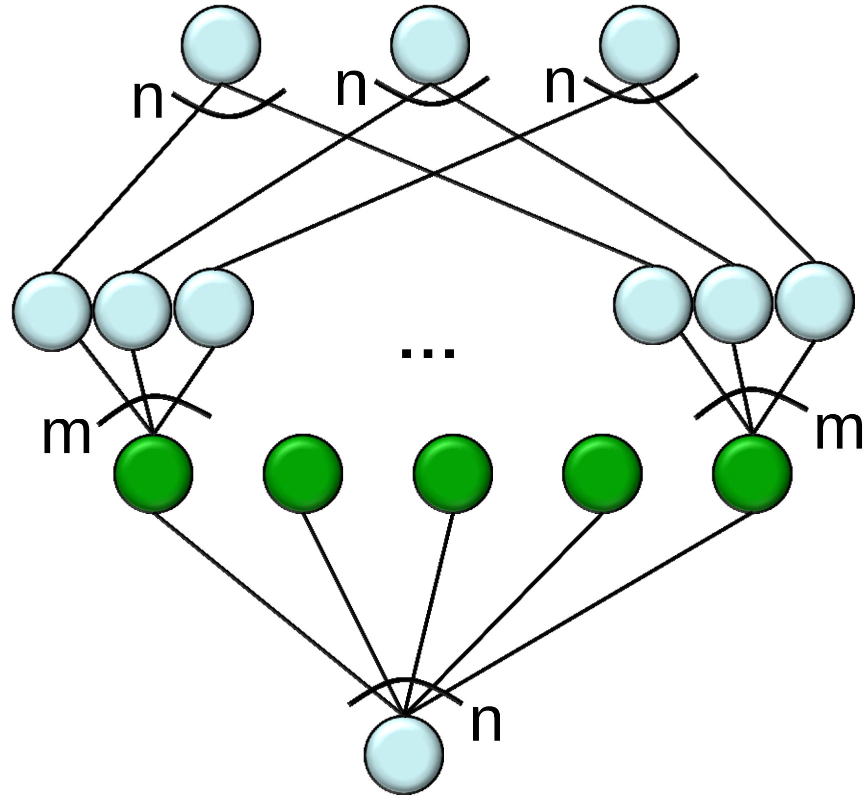


- So these green nodes must decide the same way!

Lower Bound for MDS: Intuition (3)

- But however they decide, one way will be devastating (with $n = m^2$)!





$|DS_{OPT}| = 2.$

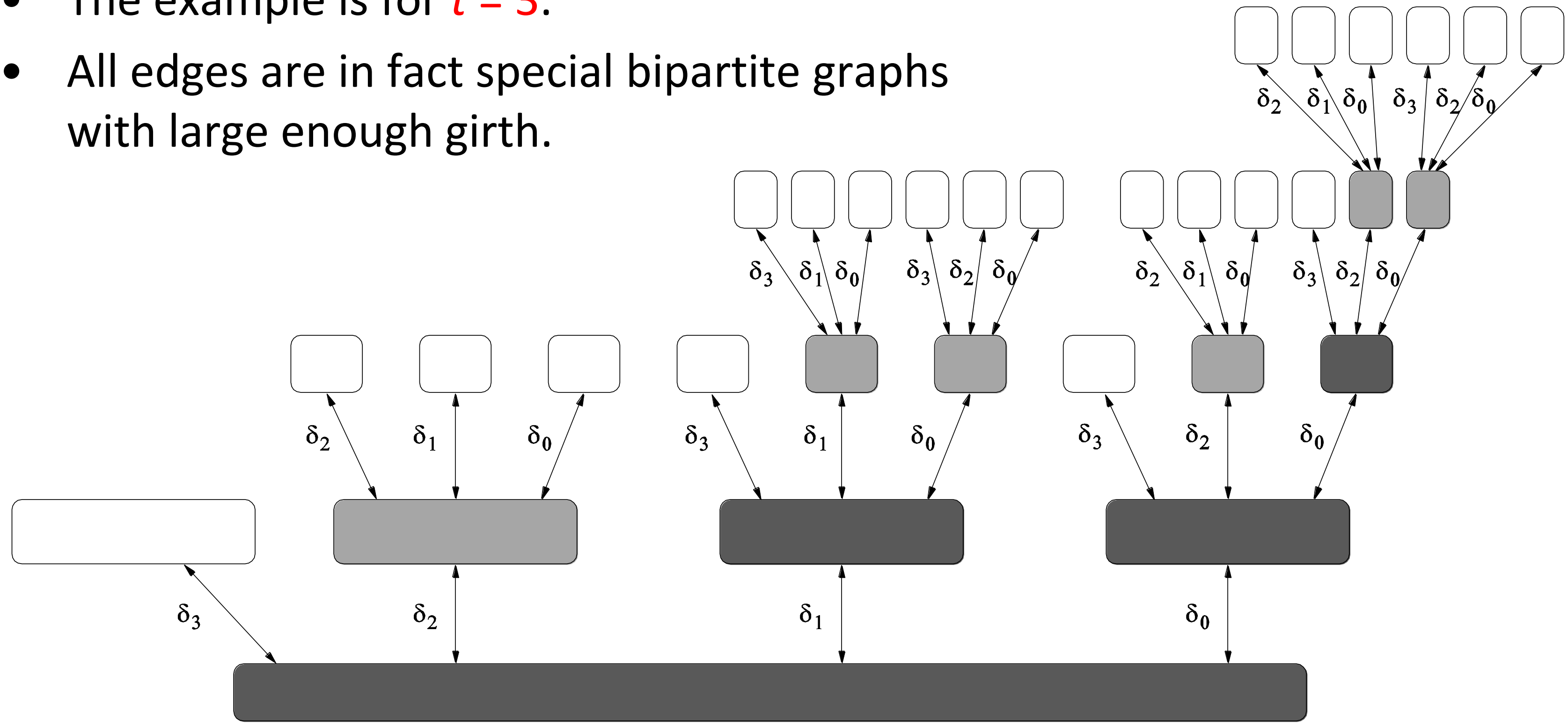
$|DS_{OPT \text{ without green}}| \geq m.$

$|DS_{OPT}| = m+1.$

$|DS_{OPT \text{ with green}}| > n$

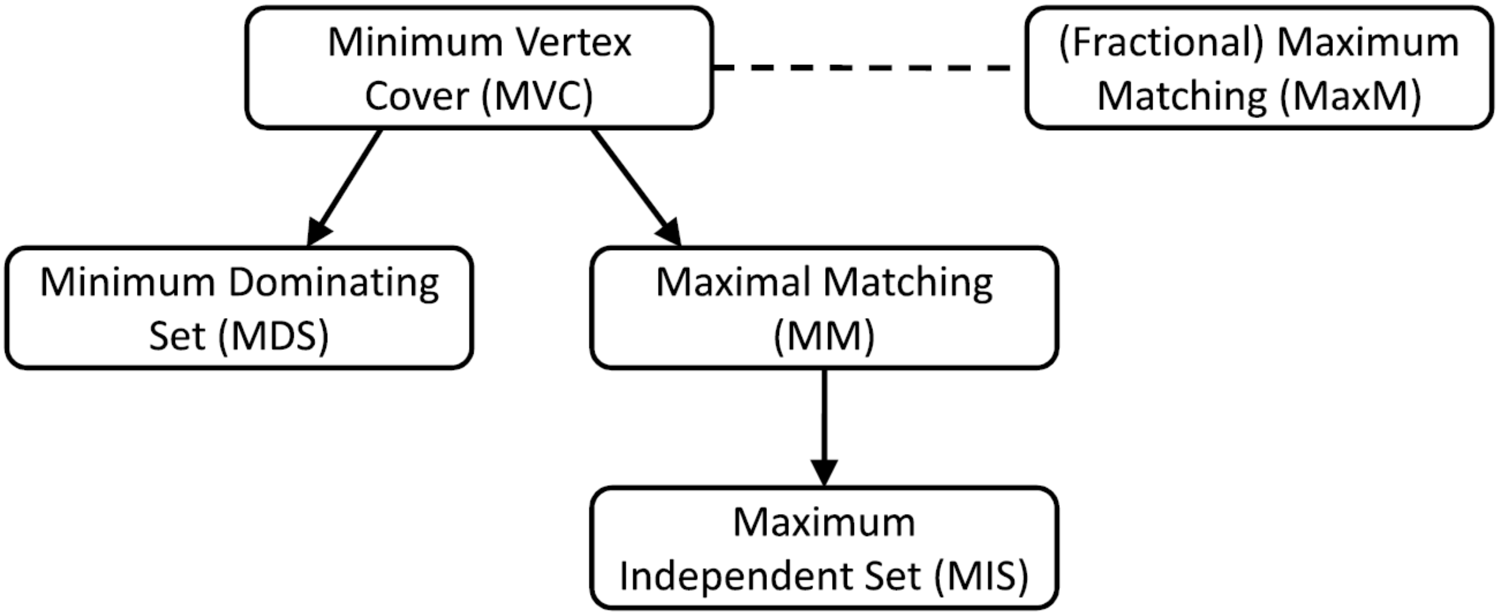
Graph Used in the Lower Bound

- The example is for $t = 3$.
- All edges are in fact special bipartite graphs with large enough girth.



Lower Bounds

- Results: Many "local looking" problems need non-trivial *t*.
- E.g., a polylogarithmic dominating set approximation (or a maximal independent set, etc.) needs at least $\Omega(\log \Delta)$ and $\Omega(\log^{1/2} n)$ time.



[Kuhn, Moscibroda, W, 2004, 2006, 2010]

Local Algorithms (“Tight” Lower & Upper Bounds)